

## 200604 - IEA - Advanced Statistical Inference

Coordinating unit:	200 - FME - School of Mathematics and Statistics
Teaching unit:	715 - EIO - Department of Statistics and Operations Research
Academic year:	2014
Degree:	MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Teaching unit Optional)
ECTS credits:	5
Teaching languages:	Spanish

### Teaching staff

Coordinator:	GUADALUPE GÓMEZ MELIS
Others:	GUADALUPE GÓMEZ MELIS - A ÀLEX SÁNCHEZ PLA - A

### Prior skills

This course is mandatory for all graduate students in statistics or mathematics.

Statistical knowledge required of an undergraduate-level in statistics or mathematics.

Basic mathematical analysis skills required: integration of functions of one or two variables, derivation, optimization of a function of one or two variables.

\* Basic probability skills required: the most common parametric distributions, properties of a normal distribution, the law of large numbers and the central limit theorem.

\* Basic statistical inference skills required: using the likelihood function for simple random sampling (independent identically distributed data), inference in the case of normality, estimation of maximum likelihood for parametric models with only one parameter and simple random sampling.

Chapters 1 through 5 from book "Statistical Inference" by Casella and Berger (2001).

### Degree competences to which the subject contributes

Specific:

3. CE-1. Ability to design and manage the collection of information and coding, handling, storing and processing it.
4. CE-4. Ability to use different inference procedures to answer questions, identifying the properties of different estimation methods and their advantages and disadvantages, tailored to a specific situation and a specific context.
5. CE-6. Ability to use appropriate software to perform the necessary calculations in solving a problem.
6. CE-9. Ability to implement statistical and operations research algorithms.

Transversal:

2. EFFECTIVE USE OF INFORMATION RESOURCES: Managing the acquisition, structuring, analysis and display of data and information in the chosen area of specialisation and critically assessing the results obtained.

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### Teaching methodology

Theory sessions of 1.5 hours

The sessions present the subject material. The teacher might use the computer to present the contents. Ideas and concepts are emphasized and a detailed look is given at those demonstrations which are pedagogical

- Chapters 1 to 5 of Gomez and Delicado will be followed. They can be downloaded from the Intranet.
- Supplementary materials will be provided for specific issues.

Problem sessions of 1.5 hours.

- With one week's notice, problems will be posted on the intranet and at the next meeting will be discussed.
- Students must come to class with the problems solved, or at least thought out.
- The professor will solve the problems and discuss with students their questions or other solutions.
- The solution of these problems will be posted after the corresponding session on the intranet.
- At the end of some sessions between 4 and 6 R problems and exercises (see below) will be proposed
- These problems must be solved individually and delivered within the period specified in class (and in the statement).
- The problems will be corrected and evaluated individually.
- The corresponding scores will weigh 20% in the final grade.

### Statistical Laboratory

R programs will serve to illustrate concepts, to complement the theoretical developments showing how statistical computing is an important tool in statistical inference.

- Some exercises will be proposed in line with those discussed in class, to strengthen the concepts.
- The exercises will be:

Resolution of minor problems

Case studies.

The corresponding scores will weigh 10% in the final grade.

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### Questionnaires

- At the end of each part of theory, a multiple choice questionnaire will be given. Students must solve this in class

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working in small groups formed at random.

- Once students have discussed the questions, they will individually delivered the answer sheet which will be used to evaluate the exercise

### Learning objectives of the subject

The Advanced Statistical Inference course provides a theoretical basis for the fundamentals of Statistics. Its main objective is to train students to think in statistical terms in order to to conduct a thorough professional habit. Also intended as a formative seed for the consolidation of young researchers in this area of science and technology while while equipping students with the resources to continue their training and making them capable to read papers published in journals of statistics.

After completing the course the student :

- \* has learnt about the different principles governing the reduction of a dataset and the different philosophies that may arise to solve a problem.
- \* knows the principle of sufficiency and likelihood and know how to distinguish between them.
- \* understands that the frequentist and Bayesian philosophy are two ways to approach a problem, not necessarily conflicting and sometimes complementary .
- \* be able to construct estimates ( point or interval ) using different methodologies.
- \* know to write down the likelihood function in different situations and learn different techniques to maximize it.
- \* be familiar with modern resampling techniques and view them as an approximation , either formal or well suited for use in situations where direct calculations are too complex or not available .
- \* will have acquired formal knowledge of the properties of estimators and hypothesis tests so thta will be able to choose the best of inferential methods in each case.

### Study load

Total learning time: 125h	Hours large group:	30h	24.00%
	Hours medium group:	0h	0.00%
	Hours small group:	15h	12.00%
	Guided activities:	0h	0.00%
	Self study:	80h	64.00%

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### Content

#### 1. introduction

Degree competences to which the content contributes:

Description:

- What is statistical inference. Philosophies of inference.
  - Concept of random variable, distribution function, expectation and variance. Moment generating function. Random vectors. Independence.
  - Simple random samples. Statistical models. Sums of random variables. Sampling from a Normal law.
- Approximations: SLLN, CLT.

#### 2. Point estimate 1: Methods to find estimators

Degree competences to which the content contributes:

Description:

- The empirical distribution function. Glivenko-Cantelli Theorem.
- Principle of substitution. The method of moments. Introduction to bootstrap.
- Likelihood Principle and the likelihood function.
- The maximum likelihood estimators. Invariance property.
- Bayesian Inference. Priors and a posteriori. Conjugate families. Loss function. Bayes Estimators.
- Estimates of bootstrap and jackknife variance estimator. Properties.

#### 3. Point estimate 2: Evaluation of estimates

Degree competences to which the content contributes:

Description:

- Mean squared error, bias, relative efficiency.
- Sufficient Statistics and the principle of sufficiency.
- Best unbiased estimator. Fisher information. Cramer-Rao theorem.
- Rao-Blackwell theorem. Lehmann-Scheffé theorem.
- Consistency. Asymptotic normality. Delta method. Asymptotic relative efficiency.
- Asymptotic theory for maximum likelihood estimator.

#### 4. Hypothesis Testing

Degree competences to which the content contributes:

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### Description:

- Basic Definitions. Neyman-Pearson Lemma.
- Uniformly more powerful tests. Monotone likelihood ratio.
- Unbiased Tests. Locally powerful test.
- Procedures based on the likelihood ratio:
  - Likelihood ratio test. Wilks theorem;
  - Score test. Wald Test. Testing parameters in the presence of "nuisance".
- Bayesian methods:
  - Bayes Factors.

## 5. Confidence regions

Degree competences to which the content contributes:

### Description:

- Limits of confidence intervals and regions.
- Duality between confidence regions and hypothesis tests.
  - Bayesian Intervals
- Bootstrap methods
  - bootstrap confidence intervals
  - permutation tests and bootstrap tests of significance.

## Qualification system

Each topic is assessed by individual take-home problems and take home exercises with R ("PRA") and a questionnaire (Q) type test which is discussed in small groups in class time. The final examination (EF) consists of resolution of problems. The final grade for the course (N) is obtained from the grades of the exercises (PRA), quizzes (Q) and final exam (EF) following the formula:

$$N = \max(EF, 0.3 * PRA + 0.2 * Q + 0.5 * EF).$$

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### Bibliography

#### Basic:

- Casella, G.; Berger, Roger L. Statistical inference. Pacific Grove Duxbury, 2002.
- Cox, D.R. Principles of statistical inference. Cambridge Univ Press, 2006.
- Wasserman, Larry. All of statistics : A concise course in statistical inference. Pittsburgh: Springer, 2004. ISBN 9781441923226.
- Garthwaite, Paul H.; Jolliffe, Ian T.; Jones, B. Statistical inference. 2nd ed. Oxford University Press, 2002.
- Azzalini, Adelchi. Statistical inference based on the likelihood. Chapman & Hall, 1996.
- Gómez Melis, G.; Delicado, P. Inferència i decisió apunts. Servei de fotocòpies, 2003.
- Ruiz-Maya Pérez, L. ; Martin Pliego, F.J. Estadística. II, inferencia. 2ª ed. Madrid: Alfa Centauro, 2001. ISBN 8472881962.

#### Complementary:

- Boos, D.D.; Stefanski, L.A. Essential statistical inference : theory and methods. Springer, 2013.
- Shao, Jun. Mathematical statistics. 2nd ed. Springer Texts in Statistics, 2003.
- Young, G.A.; Smith, R.L. Essentials of statistical inference. Cambridge University Press, 2010. ISBN 978-0521548663.