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34966 - VD - Differentiable Manifolds

Coordinating unit: 200 - FME - School of Mathematics and Statistics

Teaching unit: 749 - MAT - Department of Mathematics

Academic year: 2016

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)

ECTS credits: 7,5 Teaching languages: English

Teaching staff

Coordinator: MIGUEL ANDRES RODRIGUEZ OLMOS

Others: Primer quadrimestre:
EVA MIRANDA GALCERÁN - A
MIGUEL ANDRES RODRIGUEZ OLMOS - A

Prior skills

Basic courses on algebra, calculus, topology and differential equations, and calculus on manifolds. Students from the FME are supposed to have taken "Varietats Diferenciables" (optional 4th year course).

This is not a basic course and the students are assumed to have attended previous courses on differential geometry and smooth manifolds. Students feeling that they may not fulfill the requisites are invited to discuss their case with the lecturers. It is totally possible for prospective students with a lesser knowledge in these topics to follow this course provided they are willing to make up for the gap with individual work during the course and/or by reading some recommended bibliography prior to the beginning of the course.

Degree competences to which the subject contributes

Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
9. EFFECTIVE USE OF INFORMATI0N RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

34966 - VD - Differentiable Manifolds

Teaching methodology

Theory classes and tutorial sessions will be used to present and develop the contents of the course. Along the course the students will be given problems to solve as homework.

Learning objectives of the subject

The subject focuses on some of the fundamental topics of differential geometry and its applications in different areas, as geometric mechanics, control theory, classic and quantum field theory, fluid mechanics, computer vision, geophysical dynamics, general relativity and more.

By the end of the course, students should be able to:
- understand all the ideas developed along the course.
- apply the studied concepts to other areas of pure mathematics, physics and engineering.
- integrate in a research group on these kinds of topics and their applications.
- search and understand the scientific literature on the subject.
- write and present an essay on mathematics.

Study load

Total learning time: 187h 30m	Hours large group:	60h	32.00%
	Self study:	127h 30m	68.00%

34966 - VD - Differentiable Manifolds

Content

Reminder of Manifold Theory and Exterior Calculus	Learning time: 12h 52m Theory classes: 4h Self study : 8h 52m

Description:
Brief survey of manifold theory and differential geometry. Manifolds, atlases, smooth maps, tangent vectors and vector fields, flows, exterior calculus.

De Rham Cohomology and Integration Theory	Learning time: 25h Theory classes: 8h Self study : 17h

Description:
We define De Rham cohomology and compare to other cohomologies. We will also introduce De Rham computation kit and Poincaré duality.

Symplectic and Poisson Geometry	Learning time: 43h 45m Theory classes: 14h Self study : 29h 45m

Description:
Introduction to symplectic and Poisson manifolds with emphasis on examples. Starting with symplectic manifolds, we will explain Moser's trick and some applications to normal form theorems such as the Darboux theorem or the Lagrangian neighbourhood theorem. Special attention will be given to examples provided by the realm of integrable systems. We end the chapter introducing the basic concepts in Poisson geometry.

Lie groups and Lie algebras. Actions on Manifolds	Learning time: 25h Theory classes: 8h Self study : 17h

Description:
Introduction to the main aspects of the theory of Lie groups and their actions on manifolds, including classic groups, subgroups, actions, orbits and quotients.

Principal Bundles	Learning time: 18h 45m Theory classes: 6h Self study : 12h 45m
Description: The concept of fibre bundes and local triviality will be introduced. Then we define the main object of study, principal bundles and their main example, frame bundles, as well as their properties.	

Connections and Curvature	Learning time: 18h 45m Theory classes: 6h Self study : 12h 45m
Description: We introduce connections on principal bundles and study their existence and main constructions and properties, as curvature, holonomy, parallelism and structure equations.	

Vector Bundles and Associated Bundles	Learning time: 18h 45m Theory classes: 6h Self study : 12h 45m
Description: We will study constructions in bundle theory, as associated and pullback bundles, and the theory of general vector bundles. The main objective is to introduce connections on vector bundles and their properties, as well as their relationship with connections on principal bundles.	

Qualification system

There will be a final exam, as well as the possibility to write an optional essay that would contribute to the final grade. Students would choose, together with the lecturers, a topic that complements or advances the material taught during the course, according to their mathematical interests.

Regulations for carrying out activities

The final grade awarded to the student would we computed as follows:

- Case A: an student that does only the final exam. Then the final grade would be that of the exam.
- Case B: an student that does the final exam AND submits a written essay. Then the final note would be the result of MAX(exam, 60% exam + 40% essay)

Bibliography

Basic:

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Duistermaat, J. J. ; Kolk, Johan A. C. Lie groups. Berlin: Springer-Verlag, 2000. ISBN 3540152938.

Greub, W. H.; Halperin, S.; Vanstone, R. Connections, curvature and cohomology (vol. I). New York: Academic Press, 1972-1976.

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Cannas da Silva, Anna. Lectures on symplectic geometry. Springer-Verlag, 2008.

Tu, Loring W. An Introduction to manifolds. 2nd ed. New York: Springer, cop. 2011. ISBN 9781441973993.

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Guillemin, Victor; Pollack, Alan. Differential topology. Reprint of the 1974 original. AMS Chelsea Publishing,

Warner, Frank W. Foundations of differentiable manifolds and Lie groups. New York [etc.]: Springer, cop. 1983. ISBN 9780387908946.

Complementary:

Bott, Raoul; Tu, Loring W. Differential forms in algebraic topology. New York: Springer-Verlag, 1982. ISBN 0387906134.

Audin, Michèle. Torus actions on symplectic manifolds. 2nd ed. Progress in Mathematics, 93.Birkhäuser Verlag, 2004.

Lee, John M. Riemannian manifolds : an introduction to curvature [on line]. New York: Springer, 1997 [Consultation: 05/06/2012]. Available on: <http://link.springer.com/book/10.1007%2Fb98852>. ISBN 038798271X.

Massey, William S. Algebraic topology: an introduction. New York: Springer-Verlag, cop. 1977. ISBN 0387902716.

Warner, Frank W. Foundations of differentiable manifolds and lie groups. New York, NY [etc.]: Springer-Verlag, cop. 1971. ISBN 0387908943.

Dieudonné, Jean. Éléments d'analyse (vol. II-V). Paris: Gauthier-Villars, cop. 2003. ISBN 2876472155.

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Nakahara, Mikio. Geometry, topology, and physics. 2nd ed. New York [etc.]: Taylor & Francis, cop. 2003. ISBN 0750306068.

Holm, Darryl D; Schmah, Tanya; Stoica, Cristina; Ellis, David C. P. Geometric mechanics and symmetry : from finite to infinite dimensions. New York: Oxford University Press, cop. 2009. ISBN 9780199212910.

Gallier, Jean;Quaintance, Jocelyn. Notes on differential geometry and Lie groups. University of Pennsylvania, 2016.