200112 - EALG - Algebraic Structures

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Compulsory)
ECTS credits: 7.5
Teaching languages: Catalan

Teaching staff

Coordinator: FRANCESC D'ASSIS PLANAS VILANOVA
Others: Primer quadrimestre:
FRANCESC D'ASSIS PLANAS VILANOVA - M-A, M-B
BERNAT PLANS BERENGUER - M-B
ANA RIO DOVAL - M-A

Prior skills

Contents of Foundations of Mathematics: sets and maps; equivalence relations and order relations; permutations; arithmetic of integers and of polynomials; Euclidean algorithm and Bézout's identity; congruences (modular arithmetic); ...

Contents of Linear Algebra: vector space, subspace and quotient vector space; bases; matrices and matrix calculus; ...

Requirements

The first year courses Foundations of Mathematics and Linear Algebra

Degree competences to which the subject contributes

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking into account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

General:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of
In this course the student gets exposed to and learns some of the main results about the most common algebraic structures: groups, rings, fields and modules.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

Teaching methodology
Traditional: theory sessions in where the teacher presents the contents of the course, and problems sessions in which the professor shows how to solve some of the problems of the problems lists.

Learning objectives of the subject
In this course the student gets exposed to and learns some of the main results about the most common algebraic structures: groups, rings, fields and modules.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 187h 30m</th>
<th>Hours large group:</th>
<th>45h</th>
<th>24.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group:</td>
<td>0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Hours small group:</td>
<td>30h</td>
<td>16.00%</td>
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<td></td>
<td>Guided activities:</td>
<td>0h</td>
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<tr>
<td></td>
<td>Self study:</td>
<td>112h 30m</td>
<td>60.00%</td>
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## Content

<table>
<thead>
<tr>
<th>Groups</th>
<th>Learning time: 62h 30m</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 15h</td>
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<td></td>
<td>Practical classes: 10h</td>
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<tr>
<td></td>
<td>Self study: 37h 30m</td>
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**Description:**
- Groups, subgroups, lateral classes; homomorphisms, kernel and image; normal subgroups and quotient groups; isomorphism theorem; order of an element.
- Examples of groups: cyclic, dihedral, symmetric, alternate, cartesian product, semidirect product, ...
- Simple groups, simplicity of the alternate, Jordan-Hölder theorem, solvable groups.
- Action of a group on a set, stabilizers, orbits, orbits formula, actions by translation and by conjugation.
- Applications.
- p-groups and Sylow subgroups. Sylow Theorem. Applications.

<table>
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<tr>
<th>Rings</th>
<th>Learning time: 50h</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 12h</td>
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<td>Practical classes: 8h</td>
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<td>Self study: 30h</td>
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**Description:**
- Ring, multiplicative group, subring, ideal, homomorphism; integral ring, fraction field; prime and maximal ideals.
- Divisibility; units and associates; greatest common divisor and Bézout's identity; irreducibles, primes and unique factorization; factorial ring; principal ring; euclidean ring and euclidean algorithm.
- Polynomials with coefficients in a field; degree; euclidean division; rational functions; roots; derivation.
- Polynomials with coefficients in a factorial ring. Content and primitive polynomials; Gauss lemma; unique factorization; Eisenstein irreducibility criterion.
- Cyclotomic polynomials.

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<tr>
<th>Fields</th>
<th>Learning time: 50h</th>
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<tr>
<td></td>
<td>Theory classes: 12h</td>
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<td></td>
<td>Practical classes: 8h</td>
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<tr>
<td></td>
<td>Self study: 30h</td>
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**Description:**
- Field, examples, characteristic, prime field, embeddings.
- Extensions, degree, algebraic and transcendent elements, minimal polynomial, decomposition field, adjunction of elements, simple extensions, primitive element theorem.
- Finite fields: construction, properties and classification.
- Construction of regular polygons: Gauss periods and characterization of constructible polygons.
Midterm and final exam. The course mark will be the best of the following two: the final exam mark or 70% of the final plus 30% of the midterm.
Extraordinary exam will take place on July for students that failed during the regular semester.

### Qualification system

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<th>Modules</th>
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<tr>
<td><strong>Learning time:</strong> 25h</td>
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<tr>
<td>Theory classes: 6h</td>
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<tr>
<td>Practical classes: 4h</td>
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<tr>
<td>Self study: 15h</td>
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**Description:**
- Smith normal form of a matrix over a PID. Invariant factors.
- Theorem of classification of finitely generated modules over PID. Invariant factors and elementary divisors.

**Bibliography**

**Basic:**

**Complementary:**