200201 - TG - Galois Theory

Coordination unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2019
Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Teaching unit Optional)
ECTS credits: 6
Teaching languages: Catalan

Teaching staff
Coordinator: JORDI QUER BOSOR
Others: Primer quadrimestre: JORDI QUER BOSOR - M-A

Prior skills
Contents of Algebraic Structures: permutation groups, simple groups, Jordan-Hölder theorem, solvable groups, p-groups, polynomial rings, fields.

Requirements
The course Algebraic Structures of 3rd year.

Degree competences to which the subject contributes

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking into account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generic:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already known and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-
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appraisal. Choosing the best path for broadening one's knowledge.

Teaching methodology

Theory sessions where the teacher presents the contents of the course and problems sessions where the students and the professor solve the proposed problems.

Learning objectives of the subject

Basic concepts and results of Galois theory and its applications to the resolution by radicals of polynomial equations and to the geometric constructions with ruler and compass.

Study load

<table>
<thead>
<tr>
<th>Total learning time: 150h</th>
<th>Hours large group:</th>
<th>30h</th>
<th>20.00%</th>
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<tbody>
<tr>
<td></td>
<td>Hours medium group:</td>
<td>0h</td>
<td>0.00%</td>
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<tr>
<td></td>
<td>Hours small group:</td>
<td>30h</td>
<td>20.00%</td>
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<td></td>
<td>Guided activities:</td>
<td>0h</td>
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<tr>
<td></td>
<td>Self study:</td>
<td>90h</td>
<td>60.00%</td>
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### Content

<table>
<thead>
<tr>
<th><strong>Fields and extensions</strong></th>
<th><strong>Learning time:</strong> 50h</th>
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<tbody>
<tr>
<td></td>
<td><strong>Theory classes:</strong> 10h</td>
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<tr>
<td></td>
<td><strong>Laboratory classes:</strong> 10h</td>
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<tr>
<td></td>
<td><strong>Self study:</strong> 30h</td>
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**Description:**
- Extension of embeddings, $K$-embeddings, $K$-automorphisms, conjugation.

<table>
<thead>
<tr>
<th><strong>Galois Theory</strong></th>
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**Description:**
- Galois extensions, Galois group, Galois correspondence. Artin theorem and fundamental theorem of Galois theory.
- Examples: quadratic and multi quadratic extensions, cyclotomic extensions, extensions of finite fields, ... 
- Galois group of a polynomial. Examples: degrees 2, 3 and 4.
- Symmetric polynomials, discriminant, resolvent and resultant. Applications.

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<tr>
<th><strong>Applications</strong></th>
<th><strong>Learning time:</strong> 50h</th>
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**Description:**
- Resolution of the cubic and the quartic equations: formulas by Tartaglia and Cardano.
- Characterization of the polynomial equations solvable by radicals. Impossibility of resolution of the general quintic.
- Characterization of the numbers constructible by rule and compass.

### Qualification system

Every student can obtain up to 5 points by solving problems in the problem sessions and giving them in written form. Moreover, there will be a final exam. The course mark will be computed as $AC + (10 - AC) \times NF/10$, with $AC$ is the mark obtained in problem sessions and $NF$ is the mark of the final exam.
Bibliography

**Basic:**


**Complementary:**


