



## Course guides

# 200901 - SAGDM - Seminar on Algebra, Geometry and Discrete Mathematics

Last modified: 25/06/2020

**Unit in charge:** School of Mathematics and Statistics

**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2020

**ECTS Credits:** 3.0

**Languages:** English

## LECTURER

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**Coordinating lecturer:** JUAN JOSÉ RUE PERNA

**Others:** Segon quadrimestre:  
JUAN JOSÉ RUE PERNA - A

## PRIOR SKILLS

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The student must know the basics on graph theory. Additionally, it is necessary to have some knowledge on probability theory, group theory and arithmetics.

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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### Specific:

MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.

MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.

MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

## TEACHING METHODOLOGY

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This seminar will be based on the presentation (from the responsible and the students) of material in the context of the theory of expanders. This material will be taken from specialized books and research papers.

## LEARNING OBJECTIVES OF THE SUBJECT

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The main objective of the seminar is to show an area of mathematics that intersects both algebra and geometry, discrete mathematics and other related areas, such as computer science, low-dimensional geometry and probability theory, among others.

The main objective is to get the student to gain a basic knowledge of expander theory, as well as the various applications in various branches of contemporary mathematics. In this direction, the student will also be encouraged to learn to conduct technical talks in public and in the preparation of technical scientific documents.



## STUDY LOAD

Type	Hours	Percentage
Hours large group	24,0	32.00
Self study	51,0	68.00

Total learning time: 75 h

## CONTENTS

### Spectral graph Theory and expanders

**Description:**

- Spectral graph theory.
- Spectral bounds: spectral gap, Mixing Lemma, Alon-Boppana. Ramanujan graphs.
- Existence of graph expanders: Margulis construction. Probabilistic constructions.

**Full-or-part-time:** 20h

Theory classes: 6h 40m

Self study : 13h 20m

### Graph expanders and group theory

**Description:**

- Cayley graphs. Properties
- Random walks in Cayley graphs.
- Kazhan Property (T) and relation with expansion phenomena

**Full-or-part-time:** 9h 20m

Theory classes: 6h

Self study : 3h 20m

### Graf expanders and Number Theory

**Description:**

- Sum-product phenomena. Balog-Szemerédi-Gowers Theorem.
- Classical algebraic groups and quasi-random groups.
- Expansion in  $SL_2(\mathbb{F}_q)$ : Helfgott Theorem and Bourgain-Gamburd

**Full-or-part-time:** 12h

Theory classes: 6h

Self study : 6h



### Other applications of graph expanders

**Description:**

- Applications in knot theory: the Barzdin-Kolmogorov theorem
- Applications in theoretical computer science: design of concentrators
- Applications in theoretical computing: design of algorithms
- Applications in algebraic number theory: elliptic curves

**Full-or-part-time:** 12h

Theory classes: 6h

Self study : 6h

### GRADING SYSTEM

The grading of this seminar will be based on three points: (CA) Continuous evaluation, (MP) Material preparation and (PT) presentation.

(CA): will be based on the understanding of the material, as well as the meetings that will be held between the student and the responsible to prepare the student's presentation (or presentations). It will also include the fact of being active during the seminar sessions.

(MP): preparation of both the presentation, its good preparation and the summary sheet.

(PT): presentation. This will include questions from the teacher and students.

Overall grading of the seminar: 30% (CA)+20% (MP)+50% (PT)

### BIBLIOGRAPHY

**Basic:**

- Kowalski, Emmanuel. An introduction to expander graphs. Société Mathématique de France, 2019.
- Davidoff, Giuliana; Sarnak, Peter; Valette, Alain. Elementary number theory, group theory and Ramanujan graphs. London Mathematical Society, 2003.
- SHLOMO HOORY, NATHAN LINIAL, AND AVI WIGDERSON. "Expander graphs and their applications". Bulletin of the American Mathematical Society [on line]. [Consultation: 25/06/2020]. Available on: [https://www.cse.huji.ac.il/~nati/PAPERS/expander\\_survey.pdf](https://www.cse.huji.ac.il/~nati/PAPERS/expander_survey.pdf).
- Krebs, Mike; Shaheen, Anthony. Expander families and Cayley graphs : A beginner's guide. Oxford University Press, 2011.

**Complementary:**

- Lubotzky, Alex. Discrete groups, expanding graphs and invariant measures. Birkhäuser, 1994.