Course guide
200002 - AL - Linear Algebra

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: MIGUEL ANGEL BARJA YAÑEZ

Others: Primer quadrimestre:
MIGUEL ANGEL BARJA YAÑEZ - M-A, M-B
JESUS FERNANDEZ SANCHEZ - M-B
JAUME MARTÍ FARRÉ - M-A
MIQUEL ORTEGA SÁNCHEZ COLOMER - M-A, M-B

Segon quadrimestre:
MIGUEL ANGEL BARJA YAÑEZ - REF
JAUME MARTÍ FARRÉ - REF

PRIOR SKILLS

Ths student must master the knowledge of highe school mathematics and and be able to solve related exercises.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Theory classes will be used to present and develop the program.
In the problem sessions will be solved, among the exercises and problems proposed, those that are considered more illustrative. We will insist on the conceptual aspects of the subject without neglecting the most mechanical parts. During these sessions, the different strategies available to address the problems will be considered and the choice of the one that is most appropriate will be justified. In this sense, efforts will be made to encourage the active participation of students.

LEARNING OBJECTIVES OF THE SUBJECT

The general objective of the subject is to introduce students to different aspects of standard linear algebra and matrix analysis. The specific objectives of this subject are the acquisition of basic knowledge of linear algebra (matrices, systems of linear equations, vector spaces and their transformations). Specifically:

- manipulation and operations with matrices; discussion and solution of systems of linear equations;
- vector spaces; linear dependence; subspaces; bases and coordinates.
- study of linear applications; base changes; invariant subspaces; diagonalization of endomorphisms; Jordan form.
- introduction to the basic geometric notions related to the Euclidean space

Besides the asignatura has to be foundation and reference in back courses and, for this, the course also has like aims:

- enhance the student's capacity for abstraction;
- familiarize students with the development of abstract language and mathematical formalism;
- to introduce students to interdisciplinary problems that are solved with linear algebra.
- and introduce the student to the use of linear algebra as a tool to model and solve problems.

At the end of the course, the knowledge, skills and abilities that the student must acquire are as follows:

- Know how to operate with matrices. Calculate ranges and determinants. Know how to interpret matrices, operations and results in different contexts. Discuss and solve systems of linear equations. Know how to propose systems and know how to interpret their solutions.
- Recognize vector spaces, vector subspaces and linear applications.
- Know how to calculate linear dependence relations. Understand the notions of bases and dimension. Know how to calculate and change coordinates. Understand the different operations between subspaces and between vector spaces. Get acquainted with the dual space and the quotient and know how to work on it.
- Determine the kernel and image of a linear application. Calculate images and anti-images of elements and subspaces. Know how to represent matrix linear applications. Understand the relationship with systems of equations and know how to change the basis. Understand the concept of vector subspace and constraint. Understand the need to transform an array to a predetermined shape. Discuss and find the diagonal shape of an array, both in the real case and in the complex case. Know how to work with specific types of matrices.
-Know how to find the Jordan form of an endomorphism with a decomposable characteristic polynomial. Apply it to matrix calculus.
-Know applications of the diagonalization and Jordan form of an endomorphism.
-Understand the concept of scalar product and derived concepts. Know how to work in Euclidean spaces. Understand the notion of orthogonality and orthogonal projection. Know the real Spectral Theorem.

STUDY LOAD

Type	Hours	Percentage
Guided activities	7,5	4.00
Hours large group	45,0	24.00
Self study	105,0	56.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

Matrices, determinant and linear systems

Description:
Operations with matrices. Elementary matrices and transformations. Rank. Echelon forms. Linear systems. Rouché-Frobenius Theorem. Determinant. Properties. Adjoint. laplace Rule. Computation of the inverse matrix.

Full-or-part-time: 18h
Practical classes: 8h
Self study : 10h

Vector spaces

Description:
Vector space. Linear combination. Independence and generators. Bases. Steinitz Theorem. Dimension. Coordinates. Change of basis. Subspaces. Intersection, sum and direct sum. Grasmann formula. Quotient space.

Full-or-part-time: 41h
Theory classes: 10h
Practical classes: 6h
Self study : 25h

Linear maps

Description:
Linear maps. Kernel and image. Matrix of a linear map. Change of basis. Endomorphisms. Operations with linear maps and matrices. Dual spacel. Dual basis. Dual map. Quotient space and isomorphism theorem.

Full-or-part-time: 32h
Theory classes: 8h
Practical classes: 4h
Self study : 20h

Diagonalitzation

Description:
Eigenvectors and eigenvalues, characteritic polynomial, algebraic and geometric multiplicity. First decomposition theorem, diagonalitzation crteria. Annihilator polynomials, Cayley-Hamilton theorem, minimal poynomial.

Full-or-part-time: 33h
Theory classes: 8h
Practical classes: 5h
Self study : 20h

Jordan form of an endomorphis

Description:
The concept of classification and equivalent endomorphisms. Height of vectors. Second decomposition theorem. Jordan form of an endomorphism. Applications to matrix calculus.

Full-or-part-time: 16h
Theory classes: 4h
Practical classes: 2h
Self study : 10h

Euclidean Vector Space

Description:
Scalar products and Euclidean space; norm, distance, angles, orthogonal subspace, orthogonal projection. Orthonormal bases and Gram-Schmidt. Spectral theorem.

Full-or-part-time: 28h
Theory classes: 8h
Self study : 20h

GRADING SYSTEM

The evaluation of the subject will be carried out by means of a Partial Examination in the middle of the semester, a continuous evaluation and a final examination. The continuous assessment mark will be obtained from the assessment of problems solved and delivered periodically by the students.

The grade of the subject is obtained according to the formula:

Grade = max {final exam mark; 70% final exam mark + 20% partial exam mark + 10% continuous assessment; 90% final exam + 10% continuous assessment; 80% final exam + 20% partial exam}.

Additionally, there will be an extraordinary final exam in July for those who have failed. This will provide:

Grade = max {extraordinary final exam mark; 70% extraordinary final exam mark + 20% partial exam mark + 10% continuous assessment; 90% extraoridnary final exam + 10% continuous assessment; 80% extraordinary final exam + 20% partial exam}.

BIBLIOGRAPHY

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- Castellet, M. ; Llerena, I. Àlgebra lineal i geometria. 4a ed. Bellaterra: Universitat Autònoma de Barcelona. Servei de Publicacions, 2000. ISBN 847488943X.
- Jeronimo, G.; Sabia, J.; Tesauri, S. Álgebra lineal (recopilació de notes de l'autor) [on line]. Available on: http://mate.dm.uba.ar/~jeronimo/algebra_lineal/.

Complementary:
- Friedberg, Stephen H; Insel, Arnold J; Spence, Lawrence E. Linear algebra. 4th ed. Upper Saddle River: Pearson Education, cop. 2003. ISBN 0131202669.
- Poole, David. Álgebra lineal: una introducción moderna. 2004. ISBN 9706862722.
- Lay, David C; Murrieta Murrieta, Jesús Elmer; Alfaro Pastor, Javier. Álgebra lineal y sus aplicaciones [on line]. 3a ed. México: Pearson Educación, 2007 [Consultation: 21/06/2023]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=1275. ISBN 9789702609063.

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