Course guide
200102 - AR - Real Analysis

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: MARIA TERESA MARTINEZ-SEARA ALONSO

Others: Segon quadrimestre:
INMACULADA CONCEPCION BALDOMA BARRACA - M-A
PAU MARTIN DE LA TORRE - M-B
MARIA TERESA MARTINEZ-SEARA ALONSO - M-A, M-B

PRIOR SKILLS

Knowledge in Differential and Integral Calculus in one and several variables, and linear algebra.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Theory classes will consist of presentations by the lecturer of definitions, statements, demonstrations and examples. In problem sessions there will be exercises from a list.

LEARNING OBJECTIVES OF THE SUBJECT

The course has to be for the student a transition between Calculus and Mathematical Analysis. Because of that, an important goal for the student has to be to become used to the utility of abstraction and conceptual methods.

Even though the abstract and conceptual character is the most important, the calculus aspects of some parts (Fourier series, integrals depending of one parameter) have to be fully reached.

The course has to be useful as a preparation for the use of Mathematical Analysis in other courses like Ordinary Differential Equations (where uniform convergence is more used), Partial Differential Equations (where the mean square convergence is more used), Functional Analysis and Dynamical Systems (where the knowledge on function spaces is further developed) and Probablity Theory (where measure theory and Lebesgue integration are used). It can also be useful as a preparation for postgraduate courses on subjects like signal analysis or function theory.

STUDY LOAD

Type	Hours	Percentage
Hours small group	30,0	16.00
Self study	112,5	60.00
Hours large group	45,0	24.00

Total learning time: 187.5 h

CONTENTS

Topology in the space of continuous functions.

Description:
Sequences and series of functions: pointwise and uniform convergence.
Stone-Weierstrass Theorem.
Equicontinuous families.

Full-or-part-time: 48h 30m
Theory classes: 12h
Practical classes: 8h
Self study : 28h 30m

Fourier series.

Description:
Fourier series of periodic functions.
Bessel inequality and Parseval identity
Pointwise and uniform convergence.

Full-or-part-time: 48h 30m
Theory classes: 12h
Practical classes: 8h
Self study : 28h 30m

Lebesgue measure and integration in R.

Description:
Measurable sets and measurable functions.
Integration of measurable functions.
Dominated convergence. Integral calculus and integrals depending on parameters.
Lp spaces. Series de Fourier en L2

Full-or-part-time: 62h 30m
Theory classes: 15h
Practical classes: 10h
Self study : 37h 30m

GRADING SYSTEM

Exam midterm (EP, 30%) and final exam (EF, 70%). The grade of the final exam will be considered if it is larger than the average of the course (see the following formula). The maximum of all possibilities will be considered:

MAX (EF, 0.7*EF+0.3*EP)

Additionally, there will be an extraordinary exam during july for students who fail the course. In this case the continous evaluation will be not considered.

BIBLIOGRAPHY

Basic:
- Bartle, Robert Gardner. The Elements of integration and Lebesgue measure. New York: Wiley, 1995. ISBN 0471042226.
- Dalmasso, R. ; Witomsky, P. Analyse de Fourier et applications : exercices corrigés. Paris: Masson, 1996. ISBN 2225852995.
- Batlle Arnau, Carles ; Fossas Colet, Enric. Anàlisi real : apunts. 2002.
- Marsden, Jerrold E ; Hoffman, Michael J. Elementary classical analysis. 2nd ed. New York: W.H. Freeman, cop. 1993. ISBN 0716721058.
- Stein, Elias M; Shakarchi, Rami. Fourier analysis : an introduction. Princeton (N.J.): Princeton University Press, 2003. ISBN 9780691113845.

Complementary:
- Tao, Terence. An Introduction to measure theory [on line]. American Mathematical Society, 2011 [Consultation: 26/06/2023]. Available on: https://web-p-ebscohost-com.recursos.biblioteca.upc.edu/ehost/ebookviewer/ebook?sid=5ed05512-9d64-4fc8-8490-5ecfe5177173%40redis&vid=0&format=EB. ISBN 9780821869192.
- Gasquet, C. ; Witomski, P. Fourier analysis and applications : filtering, numerical computation, wavelets. New York: Springer, 1999. ISBN 0387984852.
- Rudin, Walter. Principios de análisis matemático. 3ª ed. México: McGraw-Hill, 1980. ISBN 007054235X.
- Whittaker, E.T.; Watson, G.N. A Course of modern analysis : an introduction to the general theory of infinite processes and of analytic functions : whit an account of the principal transcendental functions. 4th ed. Cambridge: Cambridge University Press, 1927. ISBN 0521067944.
- Lang, Serge. Real and functional analysis. 3rd ed. New York: Springer-Verlag, 1993. ISBN 0387940014.
- Bruna, Joaquim. Anàlisi real. Bellaterra: Universitat Autònoma de Barcelona, 1996. ISBN 8449006929.
- Priestley, H. A. Introduction to integration. Oxford: Clarendon Press, 1997. ISBN 0198501234.