

Course guide

200111 - AMG - Multilinear Algebra and Geometry

Last modified: 13/05/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: CARLES PADRO LAIMON

Others: Primer quadrimestre:
JOSE BURILLO PUIG - M-A
CARLES PADRO LAIMON - M-A, M-B

PRIOR SKILLS

Students must have achieved the objectives of Linear Algebra and Euclidean and Affine Geometry

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:

4. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
5. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
6. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
7. CG-1. Show knowledge and proficiency in the use of mathematical language.
8. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
9. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
10. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:

11. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Teaching hours are divided into three sessions of theory and two sessions of problem solving. In the theory lectures, the contents of the subject are explained, together with examples and proofs. In the problem solving sessions, solutions to the proposed problems are presented and discussed with the students.



LEARNING OBJECTIVES OF THE SUBJECT

To acquire a deeper knowledge on the dual space. To know the basic notions on multilinear algebra: tensors and tensor product, bases of tensor spaces, operations with tensors.

To get knowledge and skills on projective geometry: reference systems, relation with affine geometry, projectivities, classification of conics and quadrics.

To get used to synthetic and analytic solutions of geometric problems.

To be initiated to abstraction in geometry and the Erlangen program.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Self study	112,5	60.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

Multilinear algebra

Description:

- Bilinear and quadratic forms.
- The vector space of tensors.
- Tensor product. Basis.
- Symmetric tensors. Skewsymmetric tensors. Operators.
- Exterior product. Basis.

Full-or-part-time: 18h

Theory classes: 11h

Practical classes: 7h

Jordan canonical form

Description:

We continue the analysis of square matrices begun in Linear Algebra studying the Jordan canonical form.

Full-or-part-time: 5h

Theory classes: 3h

Laboratory classes: 2h



Projective geometry

Description:

- Projective space (real and complex).
- Interpretations of projective plane.
- Projective completion of an affine space.
- Linear varieties. Grassmann.
- Reference systems and projective coordinates. Equations of linear varieties.
- Cross ratio.
- Duality.
- Pappus and Desargues' theorems.
- Axiomàtic definition of the projective plane. Non desarguesian planes.

Full-or-part-time: 19h 10m

Theory classes: 11h 40m

Practical classes: 7h 30m

Quadrics

Description:

- Hyperquadrics of a projective space.
- Polarity.
- Projective classification of quadrics (real and complex).
- Affine classification of quadrics (real and complex).
- Conics. Steiner's theorem.
- Euclidean geometry inside projective geometry.

Full-or-part-time: 17h

Theory classes: 10h

Practical classes: 7h

-Projectivities

Description:

- Projectivities and homographies. Properties.
- The Main Theorem of Projective Geometry.
- Matrices of projectivities.
- Projectivities, linear varieties and duality.
- Fixed points and fixed varieties.
- Some families of projectivities: perspectivities, involutions and homologies. Poncelet's Theorem.
- Homographies of the line and the plane.
- Affine maps as projectivities

Full-or-part-time: 18h

Theory classes: 11h

Laboratory classes: 7h

GRADING SYSTEM

The final mark of the subject will be obtained from a final exam (EF mark) and a midterm exam (EP mark).

The final exam will consist of a part with some problems and a theoretical part.

The final subject mark will be the result of $\max \{ EF, 0.8 EF + 0.2 EP \}$

An extra exam will take place on July for students that failed during the regular semester.



BIBLIOGRAPHY

Basic:

- Casas Alvero, Eduardo. Analytic projective geometry. European Mathematical Society, 2011. ISBN 9783037191385.
- Greub, Werner Hildbert. Multilinear algebra. New York: Springer-Verlag, 1967.
- Puerta Sales, Fernando. Algebra Lineal. Barcelona: Edicions UPC, 2005.
- Reventós i Tarrida, Agustí. Geometria projectiva. Bellaterra: Servei de Publicacions UAB, 2000. ISBN 8449019788.

Complementary:

- Audin, Michèle. Geometry. Berlin: Springer, 2003. ISBN 3540434984.
- Hartshorne, Robin. Foundations of projective geometry. New York: Irish Press International, 2009. ISBN 9784871878371.
- Projective geometry : b3 course 2003 [on line]. Available on: https://people.maths.ox.ac.uk/hitchin/files/LectureNotes/Projective_geometry/Chapter_1_Projective_geometry.pdf.- Santaló, Luís. Geometria projectiva. 3a ed. Buenos Aires: Eudeba, 1977.
- Xambó Descamps, Sebastián. Geometria [on line]. 2a ed. Barcelona: Edicions UPC, 2001 [Consultation: 21/05/2020]. Available on: <http://hdl.handle.net/2099.3/36176>. ISBN 8483015110.