Course guide
200112 - EALG - Algebraic Structures

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2023 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: ANA RIO DOVAL

Others: Primer quadrimestre:
ANNA DE MIER VINUÉ - M-B
JORDI GUARDIA RUBIES - M-A
ANA RIO DOVAL - M-A, M-B

PRIOR SKILLS

Contents of Foundations of Mathematics: sets and maps; equivalence relations and order relations; permutations; arithmetic of integers and of polynomials; Euclidean algorithm and Bézout's identity; congruences (modular arithmetic); ...
Contents of Linear Algebra: vector space, subspace and quotient vector space; bases; matrices and matrix calculus; ...

REQUIREMENTS

The first year courses Foundations of Mathematics and Linear Algebra

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
9. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

During the theory sessions, the teacher presents the contents of the course. In the laboratory sessions, with fewer students, some problems and practical activities will be worked out.

LEARNING OBJECTIVES OF THE SUBJECT

In this course the student gets exposed to and learns some of the main results about the most common algebraic structures: groups, rings, fields and modules.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Hours small group	30,0	16.00
Self study	112,5	60.00

Total learning time: 187.5 h

CONTENTS

Rings

Description:
Basic notions on rings. Ideals. Integral domains. Field of fractions. Factorial, principal and euclidean rings. Polynomial rings. Modular rings. RSA cryptosystem. Quadratic rings.

Full-or-part-time: 62h 30m
Theory classes: 15h
Practical classes: 10h
Self study : 37h 30m

Fields

Description:
Basic notions on fields. Elementary examples. Algebraic and transcendental extensions. Number fields. Primitive element theorem. Algebraic closure of a field. Finite fields and their applications. Cyclotomic fields. Ruler and compass constructions. Origami constructions.

Full-or-part-time: 62h 30m
Theory classes: 15h
Practical classes: 10h
Self study : 37h 30m

Groups

Description:
Basic concepts on groups. Classical examples of groups. Action of a group on a set. Sylow subgroups. Representations of groups. Discrete logarithm

Full-or-part-time: 37h 30m
Theory classes: 9h
Practical classes: 6h
Self study : 22h 30m

Modules

Description:
Elementary concepts on modules. Finitely generated modules over principal ideal domains. Applications.

Full-or-part-time: 25h
Theory classes: 6h
Practical classes: 4h
Self study : 15h

GRADING SYSTEM

Along the course we will make some assessed activities, representing the 15% of the final grade of the course. A midterm exam (35%) and a final exam (50%) will complement these activities to yield the final grade. If the final exam grade is higher than this weighted mean, the final grade will be that of the exam.

Students failing the course have an extraordinary exam at the end of the academic year.

BIBLIOGRAPHY

Basic:
- Garrett, P.B. Abstract algebra [on line]. Boca Raton: Chapman & Hall/CRC, 2008 [Consultation: 26/06/2023]. Available on: http://www.math.umn.edu/~garrett/m/algebra/Whole_with_TOC.pdf. ISBN 9781584886891.
- Lee, Gregory T. Abstract algebra [on line]. Springer, 2018Available on: https://dokumen.pub/abstract-algebra-an-introductory-course-3319776487-9783319776484.html. ISBN 9783319776484.
- Fraleigh, John B. A First course in abstract algebra. 7th ed. Essex: Pearson Education, 2014. ISBN 9781292024967.
- Paulsen, W. Abstract algebra : an interactive approach [on line]. CRC Press, 2016 [Consultation: 26/06/2023]. Available on: https://www-taylorfrancis-com.recursos.biblioteca.upc.edu/books/mono/10.1201/9781315370972/abstract-algebra-william-paulsen. ISBN 9781498719773.

Complementary:
- Allenby, R. B. J. T. Rings, fields and groups : an introduction to abstract algebra. London: Edward Arnold, 1983. ISBN 0713134763.
- Artin, Michael. Algebra. 2nd. Boston: Prentice-Hall, 2011. ISBN 9780132413770.
- Lang, Serge. Algebra. 3rd ed. New York: Springer, 2002. ISBN 038795385X.
- Hungerford, T.W. Algebra. New York: Springer-Verlag, 1974. ISBN 0387905189.

RESOURCES

Hyperlink:
- Expository papers by K. Conrad: https://kconrad.math.uconn.edu/blurbs/. Collection of notes by K. Conrad