Course guide
200141 - EDOS - Ordinary Differential Equations

Last modified: 11/04/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: JORDI VILLANUEVA CASTELLTORT

Others: Primer quadrimestre:
JEZABEL CURBELO HERNANDEZ - M-A, M-B
GEMMA HUGUET CASADES - M-A, M-B
JORDI VILLANUEVA CASTELLTORT - M-A, M-B

PRIOR SKILLS

Linear and multilinear algebra, differential and integral calculus, topology, physics, computer science, and one complex variable.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:
5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
9. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

There are three one hour lectures and two one hour problem sessions per week.

LEARNING OBJECTIVES OF THE SUBJECT

1) Correctly apply the fundamental theorems on ODEs as well as correctly understand and use the mathematical tools involved in their proof.
2) Solve several simple ODEs of dimension one as well as equations and linear systems with constant coefficients.
3) Know the basic concepts and basic calculation tools in relation to the qualitative study of linear and non-linear systems of ODEs, both in the autonomous and periodic case.
4) Understand the connection between the resolution of ODEs and the mathematical modeling of some simple problems of a geometric or applied nature.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Self study	112,5	60.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

First Order ODEs

Description:
Basic notions about first order ODEs and their solutions (Cauchy's problem, equation of a bundle of curves, isoclines). Changes of variables. Classic examples of ODEs solvable by quadratures (separable, linear, Bernoulli, Ricatti, homogeneous, equation of orbits, exact and integrating factors, Lagrange, Clairaut).

Full-or-part-time: 32h 30m
Practical classes: 13h
Self study : 19h 30m

Fundamental Theorems

Description:
Introduction to ODEs and basic definitions. Cauchy's problem. Theorems of existence and uniqueness of solutions (Picard and Peano). Gronwall's Lemma. Prolongation of solutions and maximal solutions. Regularity of the solutions with respect to initial conditions and parameters. Variational equations.

Full-or-part-time: 70h
Theory classes: 23h
Practical classes: 5h
Self study : 42h

Linear Equations and Linear Systems of ODEs

Description:
Systems of first-order linear ODEs (formalization and structure of solutions, homogeneous systems, matrix solutions and fundamental matrices, Liouville's formula and application to the evolution of volume by a non-linear system, non-homogeneous systems, solution of systems with constant coefficients, parameter variation formula). Linear ODEs of order n (formalization and reduction to first-order systems, method of order reduction, resolution of linear ODEs with constant coefficients, parameter variation method, method of indeterminate coefficients, oscillations). Periodic linear systems (Floquet's theorem, monodromy matrix, multipliers and characteristic exponents, introduction to the concept of stability).

Full-or-part-time: 52h 30m
Theory classes: 12h
Practical classes: 9h
Self study : 31h 30m

Qualitative Theory of ODEs

Description:
Critical points and periodic orbits. Phase portrait. Stability in the sense of Lyapunov. Stability of linear systems at constant coefficients. Stability of fixed points of nonlinear systems (linearization and method of Lyapunov functions). Poincaré application. Stability by linearization of periodic orbits. Equivalence and conjugation of vector fields. Tubular flow theorem. Hartman's theorems.

Full-or-part-time: 32h 30m
Theory classes: 10h
Practical classes: 3h
Self study : 19h 30m

GRADING SYSTEM

A partial exam (P), and a final exam (F). The final grade is N = max(F, 0.3*P+0.7*F).

An extra exam will take place on July for students that failed during the regular semester.

EXAMINATION RULES.

Students can use a handwritten sheet of paper (DIN A4 size), except in the theoretical part of the exams.

BIBLIOGRAPHY

Basic:

- Arnol'd, Vladimir Igorevich. Ordinary differential equations. Cambridge (Massachusetts): The Mit Press, 1973. ISBN 0262010372.
- Braun, Martin. Differential equations and their applications. 4a ed. Springer-Verlag, 1993. ISBN 0387978941.
- Sotomayor, Jorge. Liçoes de equaçóes diferenciais ordinárias. Rio de Janeiro: Instituto de Matemática Pura e Aplicada (IMPA), 1979. ISBN 9216050624.
- Meiss, J.D. Differential dynamical systems [on line]. 2007. Philadelphia: Society for Industrial & Applied Mathematics, 2007 [Consultation: 21/06/2023]. Available on: http://epubs.siam.org/doi/book/10.1137/1.9780898718232. ISBN 9780898716351.
- Strogatz, Steven H. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering [on line]. 2a ed. CRC Press, 2018 [Consultation: 15/06/2023]. Available on: https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=1181622. ISBN 0813349109.
- Tenenbaum, Morris; Pollard, Harry. Ordinary differential equations : an elementary textbook for students of mathematics, engineering, and the sciences [on line]. New York: Dover Publications, 1985 [Consultation: 21/06/2023]. Available on: https://www.simiode.org/resources/2942. ISBN 0486649407.
- Teschl, Gerald. Ordinary differential equations and dynamical systems [on line]. Providence: Amer. Math. Soc, 2012 [Consultation: 21/06/2023]. Available on: https://www.mat.univie.ac.at/~gerald/ftp/book-ode/. ISBN 9780821883280.