



Course guide

200142 - EDPS - Partial Differential Equations

Last modified: 11/04/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Compulsory subject).

Academic year: 2024 **ECTS Credits:** 7.5 **Languages:** Catalan

LECTURER

Coordinating lecturer: XAVIER CABRE VILAGUT

Others: Segon quadrimestre:
XAVIER CABRE VILAGUT - M-A, M-B
ALBERT MAS BLESA - M-A, M-B

PRIOR SKILLS

Those obtained in the subjects already carried out in the Degree.

REQUIREMENTS

Those obtained in the subjects already carried out in the Degree.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:

1. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
2. CE-3. Have the knowledge of specific programming languages and software.
3. CE-4. Have the ability to use computational tools as an aid to mathematical processes.

Generical:

5. CB-1. Demonstrate knowledge and understanding in Mathematics that is founded upon and extends that typically associated with Bachelor's level, and that provides a basis for originality in developing and applying ideas, often within a research context.
6. CB-2. Know how to apply their mathematical knowledge and understanding, and problem solving abilities in new or unfamiliar environments within broader or multidisciplinary contexts related to Mathematics.
7. CB-3. Have the ability to integrate knowledge and handle complexity, and formulate judgements with incomplete or limited information, but that include reflecting on social and ethical responsibilities linked to the application of their knowledge and judgements.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
9. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Theory classes with the exposition of new concepts and review of others already studied in previous subjects. They will consist of presentations by the teacher of the statements, proofs, and examples. In problem classes: problem solving of a collection previously proposed to the student. Among the objectives of the course, problem solving will have a good weight, some of them promoting and prioritizing the intuition and creativity of the student.

LEARNING OBJECTIVES OF THE SUBJECT

- To know how to calculate with the methods of separation of variables and Fourier series and with the method of fundamental solutions.
- To know both the maximum principle and its consequences and the method of integral calculation (energy, Dirichlet's principle) and consequences.
- To know the relationship between the Laplacian and the heat equation with random paths, the discrete Laplacian, the probability densities and the Gaussian. Here the abstract and conceptual character will be a priority.
- To know how to calculate with the characteristics method.
- The subject must serve to review and consolidate many concepts of Calculus and Mathematical Analysis learned by the student in previous subjects. Due to the large number of tools used by the theory of EDPs, concepts learned will also be reviewed in other compulsory subjects: complex variable, EDOs, Probability, Numeric.
- The course must also serve to motivate and prepare postgraduate or elective courses, such as Functional Analysis, Financial Mathematics and Numeric for EDPs.

STUDY LOAD

Type	Hours	Percentage
Hours large group	45,0	24.00
Self study	112,5	60.00
Hours small group	30,0	16.00

Total learning time: 187.5 h

CONTENTS

First order equations

Description:

The linear transport equation: travelling waves, characteristics, stability. The non homogeneous equation and Duhamel's formula.

Quasilinear first order equations: method of the characteristics. Examples: traffic dynamics, Burgers equation.

Full-or-part-time: 36h

Theory classes: 6h

Practical classes: 7h 30m

Self study : 22h 30m



Banach spaces, operators, and semigroups

Description:

Review of the fundamental concepts and properties of Banach spaces and the linear maps on them.

Concepts of operators and semigroups appeared in the previous chapter.

Full-or-part-time: 36h

Theory classes: 7h 30m

Practical classes: 6h

Self study : 22h 30m

The wave equation

Description:

The equation of the vibrating string: derivation; d'Alembert formula; non homogeneous equations; domains of dependence and of influence; propagation and reflection of waves; energy.

Classification of linear 2nd order PDEs: canonical form.

Full-or-part-time: 36h

Theory classes: 7h 30m

Practical classes: 6h

Self study : 22h 30m

The diffusion or heat equation

Description:

The diffusion equation in bounded domains: separation of variables and Fourier series; energy method and uniqueness; maximum principle and uniqueness.

The diffusion equation in \mathbb{R}^n : fundamental solution; the Dirac delta; convolution; existence and uniqueness theorem; regularity; non homogeneous equations and Duhamel principle.

The diffusion equation from random walks: random walk and propagation of errors; relation between caloric functions and probability densities and the Gaussian distribution.

Full-or-part-time: 36h

Theory classes: 7h 30m

Practical classes: 6h

Self study : 22h 30m



The Laplace and Poisson equations

Description:

Properties of harmonic functions: examples; separation of variables and Poisson equation in a ball; mean value property, maximum principle and uniqueness; Harnack and Liouville properties; relation between harmonic functions, random walks, the discrete Laplacian and exit probabilities.

Fundamental solution and Green function: Newtonian potential; Green function; reflection method: Green function for the half-space and the ball.

Dirichlet minimization principle and the energy method.

Full-or-part-time: 36h

Theory classes: 7h 30m

Practical classes: 6h

Self study : 22h 30m

GRADING SYSTEM

First there will be a midterm exam (P). At the end of the term there will be a final exam (F). The final subject mark will be the maximum between F and $(0,5 \cdot P + 0,5 \cdot F)$.

An extra exam will take place after the Final exam for those students who failed during the regular semester.

EXAMINATION RULES.

In the exams any kind of material, class notes or formularies will be forbidden. The midterm exam does not eliminate topics for the final exam.

BIBLIOGRAPHY

Basic:

- Salsa, Sandro. Partial differential equations in action : from modelling to theory [on line]. Milan: Springer, 2008 [Consultation: 27/06/2023]. Available on : <https://ebookcentral-proquest-com.recursos.biblioteca.upc.edu/lib/upcatalunya-ebooks/detail.action?pq-origsite=primo&docID=3062992>. ISBN 9788847007512.
- Pinchover, Yehuda; Rubinstein, Jacob. Introduction to partial differential equations. Cambridge: Cambridge University Press, 2005. ISBN 978052161323X.
- Strauss, W.A. Partial differential equations : an introduction. 2nd ed. New York: Wiley, 2008. ISBN 9780470054567.
- Shearer, Michael; Levy, Rachel. Partial differential equations : an introduction to theory and applications. Princeton: Princeton University Press, [2015]. ISBN 9780691161297.

Complementary:

- Peral, Ireneo. Primer curso de ecuaciones en derivadas parciales. Argentina: Addison-Wesley, 1995. ISBN 0201653575.