Course guide
200202 - TOPA - Algebraic Topology

Last modified: 01/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan

LECTURER

Coordinating lecturer: MARIA IMMACULADA GALVEZ CARRILLO

Others: Segon quadrimestre:
MARIA IMMACULADA GALVEZ CARRILLO - M-A

PRIOR SKILLS

The abilities acquired in the Topology module.
The abilities acquired in the Affine and Euclidean Geometry module.
The abilities acquired in the Algebraic Structures module.
The user and programmer abilities with diverse software platforms used in several modules for symbolic and numerical calculus and graphical representation will be useful.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Half of the class time will be devoted to lectures about the theoretical contents of the module, delivered by the lecturer. The other half will be devoted to the discussion and solution of problems related to these contents by the lecturer and the students, and also to the presentation of specific enhancement tasks chosen and developed by the students among the offer made available by the lecturer subject to agreement with her.

LEARNING OBJECTIVES OF THE SUBJECT

* To be acquainted with singular homology and its calculation (by means of generators and relations) for a wide range of topological spaces and variants of the theory, by hand and by computer.
* To be acquainted with several geometrical applications of singular homology.
* To be acquainted with the fundamental group as an essential tool to study topological spaces, also by understanding its relation with the first homology group, and by using the basic category tools which are crucial to its construction.
* To be acquainted with the computation of the fundamental group for a wide range of topological spaces and versions, by hand and by computer.
* To be acquainted with the concept of topological manifold for a general finite dimension, and in this context, with those of local homology, orientations, and with the dimension theorem.
* To be acquainted with applications of homology and homotopy theory, both theoretical (in the context of several areas of mathematics) and applied (image recognition, topological data analysis, shape theory...).
* To understand the intertwinning between different areas of mathematics and , in particular, to understand how topological problems can be solved by algebraic means and viceversa.

STUDY LOAD

Type	Hours	Percentage
Hours large group	30,0	20.00
Self study	90,0	60.00
Hours small group	30,0	20.00

Total learning time: 150 h

CONTENTS

Course Syllabus

Description:
Section 1: Algebraic preliminaries

Finitely generated abelian groups and the classification algorithm.
Abelian groups: short exact sequences.
Abelian groups: complexes.
Morphisms of complexes.
Homology of complexes.
Homotopies between morphisms of complexes.
Homology long exact sequence.
Non abelian groups, centre, commutator, free groups, free product of groups.

Big group: 2h, Medium group: 4h, Self-learning: 6h

Section 2: Singular Homology

Singular chain complex of a topological space. Singular homology of a topological space.
Functoriality.
H_0 and arc-connectedness.
Continuous maps homotopies.
Homotopical invariance.
Small chains theorem.
Mayer-Vietoris long exact sequence.
The singular homology of the spheres.
Relative singular homology.
Excision Theorem.
Applications: separation and non-separation theorems.

Big group: 10h, Medium group: 13h, Self-Learning: 23h

Section 3: The Fundamental Group

The fundamental group of a topological space.
Functoriality. Introduction to category theory.
Homotopical invariance. Introduction to higher order category theory.
The fundamental group of the circle.
The Seifert-Van Kampen theorem.
The Hurewicz isomorphism.
Application: toric knots.

Big group: 10h Small group: 13h, Self-learning: 23h

Section 4: Topological manifolds

Local homology.
Dimension invariance.
Orientation.

Full-or-part-time: 120h
Theory classes: 30h
Practical classes: 30h
Self study : 60h

GRADING SYSTEM

The course mark will be obtained by application of the formula below, where the partial marks appearing are

EF= Final Exam, RP=exercises resolution, PT= Thesis Presentation

N = 0,5 RP + 0,3 PT + 0,2 EF

BIBLIOGRAPHY

Basic:
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- Kosniowski, Czes. Topología algebraica. Barcelona: Reverté, cop. 1986. ISBN 8429150986.
- Bott, R.; Tu, L. Differential forms in algebraic topology. ISBN 978144197400.
- Massey, William S. A Basic course in algebraic topology. New York: Springer-Verlag, cop. 1991. ISBN 038797430X.

Complementary:
- Munkres, James R. Elements of algebraic topology. The Benjamin/Cummings Publishing Company, 1984. ISBN 0201045869.
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- Sato, Hajime. Algebraic topology : an intuitive approach. Providence: American Mathematical Society, 1999. ISBN 0821810464.