Course guide
200204 - GA - Algebraic Geometry

Last modified: 06/06/2023

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

Academic year: 2023 **ECTS Credits:** 6.0 **Languages:** Catalan, Spanish

LECTURER

Coordinating lecturer: MIGUEL ANGEL BARJA YAÑEZ

Others: Segon quadrimestre:
MIGUEL ANGEL BARJA YAÑEZ - M-A
JESUS FERNANDEZ SANCHEZ - M-A

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
4. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
5. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
6. CG-1. Show knowledge and proficiency in the use of mathematical language.
7. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
8. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
9. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
10. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
11. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
12. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

(Section not available)

LEARNING OBJECTIVES OF THE SUBJECT

(Section not available)

STUDY LOAD

Type	Hours	Percentage
Hours small group	30,0	20.00
Self study	90,0	60.00
Hours large group	30,0	20.00

Total learning time: 150 h

CONTENTS

Algebraic plane curves

Description:
Affine and projective algebraic sets. Hilbert's Nullstellensatz. Algebraic curves. Smooth and singular points. Tangent cone. Intersection theory of plane curves. Resultant and intersection multiplicity. Bézout's theorem. Plucker formules. Cremona transformations.
Af+Bg Noether's theorem. Group structure of the smooth cubic.

Full-or-part-time: 37h 30m
Theory classes: 7h 30m
Laboratory classes: 7h 30m
Self study : 22h 30m

Riemann surfaces

Description:
Riemann surfaces. Morphisms between Riemann surfaces. Degree and ramification. Differential forms. Topological interpretation of the genus. Analytical interpretation of the genus. Desingularization of plane curves: the Riemann surface associated to a plane curve. Riemann-Hurwitz formula. Hyperelliptic curves.

Full-or-part-time: 37h 30m
Theory classes: 7h 30m
Laboratory classes: 7h 30m
Self study : 22h 30m

The Riemann-Roch theorem

Description:
Linear series and divisors. Associate divisors to a function and to a differential form. Canonical linear series: degree and dimension. Riemann-Roch theorem. Applications: ellyptic curves, low genus curves, the canonical embedding, Weierstrass points, Jacobian of a curve.

Full-or-part-time: 37h 30m
Theory classes: 7h 30m
Laboratory classes: 7h 30m
Self study : 22h 30m

GRADING SYSTEM

Work in Problem classes, projects during the term and a final work or examn. The student can request a final exam.
The qualification of the course will be based on the work done by the student in the class of problems, the elaboration of some small project during the course (continuous assessment , up to 60% of the overall mark) , and a final test , which will consist of an exam or the preparation of a project. Students may decide to perform only a final exam.

BIBLIOGRAPHY

Basic:
- Fulton, William. Curvas algebraicas : introducción a la geometría algebraica. Barcelona: Reverté, 1971. ISBN 8429150757.
- Griffiths, Phillip A. Introduction to algebraic curves. Providence: American Mathematical Society, cop. 1989. ISBN 0821845306.
- Kirwan, Frances Clare. Complex algebraic curves. Cambridge: Cambridge University Press, 1992. ISBN 0521423538.
- Gibson, Christopher G. Elementary geometry of algebraic curves : an undergraduate introduction. Cambridge: Cambridge University Press, 1998. ISBN 0521646413.
- Casas-Alvero, Eduardo. Algebraic curves, the Brill and Noether way [on line]. Springer Verlag, 2019 [Consultation: 01/06/2022]. Available on: https://link.springer.com/book/10.1007/978-3-030-29016-0. ISBN 9783030290153.

Complementary:
- Miranda, Rick. Algebraic curves and Riemann surfaces. Providence: American Mathematical Society, 1995. ISBN 0821802682.
- Ghys, Étienne. A Singular mathematical promenade. Lyon: ENS Éditions, 2017. ISBN 9782847889390.