Course guide
200248 - MNED - Numerical Methods for Differential Equations

Last modified: 11/04/2024

Unit in charge: School of Mathematics and Statistics
Teaching unit: 751 - DECA - Department of Civil and Environmental Engineering.

Degree: BACHELOR'S DEGREE IN MATHEMATICS (Syllabus 2009). (Optional subject).

Academic year: 2024 **ECTS Credits:** 6.0 **Languages:** English

LECTURER

Coordinating lecturer: ESTHER SALA LARDIES

Others: Primer quadrimestre:
SONIA FERNANDEZ MENDEZ - M-A
ESTHER SALA LARDIES - M-A

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
3. CE-1. Propose, analyze, validate and interpret simple models of real situations, using the mathematical tools most appropriate to the goals to be achieved.
4. CE-2. Solve problems in Mathematics, through basic calculation skills, taking in account tools availability and the constraints of time and resources.
5. CE-3. Have the knowledge of specific programming languages and software.
6. CE-4. Have the ability to use computational tools as an aid to mathematical processes.
7. Ability to solve problems from academic, technical, financial and social fields through mathematical methods.

Generical:
1. CB-4. Have the ability to communicate their conclusions, and the knowledge and rationale underpinning these to specialist and non-specialist audiences clearly and unambiguously.
2. To have developed those learning skills necessary to undertake further interdisciplinary studies with a high degree of autonomy in scientific disciplines in which Mathematics have a significant role.
8. CG-1. Show knowledge and proficiency in the use of mathematical language.
9. CG-2. Construct rigorous proofs of some classical theorems in a variety of fields of Mathematics.
10. CG-3. Have the ability to define new mathematical objects in terms of others already know and ability to use these objects in different contexts.
11. CG-4. Translate into mathematical terms problems stated in non-mathematical language, and take advantage of this translation to solve them.
12. CG-6 Detect deficiencies in their own knowledge and pass them through critical reflection and choice of the best action to extend this knowledge.

Transversal:
13. ENTREPRENEURSHIP AND INNOVATION: Knowing about and understanding how businesses are run and the sciences that govern their activity. Having the ability to understand labor laws and how planning, industrial and marketing strategies, quality and profits relate to each other.
14. SUSTAINABILITY AND SOCIAL COMMITMENT. Being aware of and understanding the complexity of social and economic phenomena that characterize the welfare society. Having the ability to relate welfare to globalization and sustainability. Being able to make a balanced use of techniques, technology, the economy and sustainability.
15. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
16. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
17. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.

TEACHING METHODOLOGY

Lectures will be divided into sessions in a standard classroom and sessions in a computer room. In the former, theoretical concepts will be discussed while the sessions in the computer room will be used to implement the numerical methods, to analyse their performance and to solve application examples. Some sessions will be dedicated to working on the proposed exercises.

The course information and all the material will be published on the intranet.

LEARNING OBJECTIVES OF THE SUBJECT

The course provides a solid theoretical and practical basis on numerical methods for solving ordinary differential equations (ODE) and partial differential equations (PDE). This will let students continue with courses in modelling and application of differential equations in science and engineering.

By the end of the course, students should have acquired:
- Familiarization with Runge-Kutta and linear multistep methods for solving ODEs and Finite Differences and Finite Elements methods for PDEs.
- A general overview on the most important computational aspects arising from the numerical solution of differential equations.
- Knowledge on the properties and limitation of the methods.
- Ability to understand results and control the accuracy of numerical solutions.
- Experience on the use of basic and commercial codes.

STUDY LOAD

Type	Hours	Percentage
Self study	90,0	60.00
Hours large group	30,0	20.00
Hours small group	30,0	20.00

Total learning time: 150 h

CONTENTS

1. Ordinary equations. Basic concepts. Truncation error and order of a method. Convergence

Description:
Initial and boundary value problems. Euler, enhanced Euler and implicit Euler methods. Local and global truncation error. Order of a method. Numerical estimate of the order. Convergence.

Full-or-part-time: 15h
Theory classes: 3h
Laboratory classes: 3h
Guided activities: 2h
Self study : 7h

2. Runge-Kutta and Linear Multistep methods. Implementation.

Description:
Explicit and implicit Runge-Kutta methods. Absolute stability region. Examples. Numerical analysis of stiff problems: van der Pol equation.
General properties of linear multistep methods. Methods of Adams-Bashforth and Adams-Moulton. BDF methods.
Predictor-corrector methods. Conditions of consistency, stability and convergence. Local error estimates and stepsize adaptivity.
Commercial and freeware implementations.

Full-or-part-time: 24h
Theory classes: 6h
Laboratory classes: 6h
Guided activities: 3h
Self study : 9h

3. Partial Differential Equations (PDE). Generalities on their solution

Description:
Problems in engineering and the applied sciences requiring numerical solution of PDE. Linear 2nd order PDE: classification, physical interpretation. Fundamental aspects of their numerical solution. Boundary conditions.

Full-or-part-time: 26h
Theory classes: 5h
Laboratory classes: 5h
Guided activities: 4h
Self study : 12h

4. Numerical solution of PDE with the Finite Difference Method (FDM)

Description:
Difference operators. Discretization of the unidimensional parabolic equation with the Finite Difference Method (FDM). Systems of difference equations. Analysis of convergence, stability and consistence. Multidimensional problems and applications.
Discretization with the FDM. Drawbacks when compared to the Finite Element Method (FEM).

Full-or-part-time: 26h
Theory classes: 5h
Laboratory classes: 5h
Guided activities: 4h
Self study : 12h

5. Introduction to oundary value problems. The shooting method. Other methods.

Description:
Strong form, method of weighted residuals and weak form for elliptic equations. Boundary conditions. Finite element interpolation: mesh and splines. Numerical integration. Reference element and isoparametric transformation. Frequently used elements. Efficient implementation of a finite element code. Convergence properties. Time integration in transient problems.

Full-or-part-time: 26h
Theory classes: 5h
Laboratory classes: 5h
Guided activities: 4h
Self study : 12h

6. Quality control of solutions

Description:
Need for ensuring the quality of the solution. Concepts of verification and validation. Basic concepts for error estimates, estimate of quantities of interest. Remeshing and adaptivity.

Full-or-part-time: 15h
Theory classes: 3h
Laboratory classes: 3h
Guided activities: 2h
Self study : 7h

GRADING SYSTEM

The final mark is obtained as
- 50% from continuous assessment (assignments, short deliverables... partially done in class)
- 50% from exams
All the marks are out of 10 and the passing mark is 5.

EXAMINATION RULES.

Attending a minimum of lessons and coursework are compulsory.

BIBLIOGRAPHY

Basic:
- Butcher, J. C. Numerical methods for ordinary differential equations. Wiley, 2016.
- Dekker, K.; Verwer, J.G. Stability of Runge-Kutta methods for stiff nonlinear differential equations. Elsevier, 1984. ISBN 0444876340.
- Shampine, L. F.; Gladwell, I.; Thompson, S. Solving ODEs with MATLAB. Cambridge University Press, 2003. ISBN 0521530946.
- Hoffman, Joe D. Numerical methods for engineers and scientists. 2nd. New York: Marcel Dekker, 2001. ISBN 0824704436.
- Quarteroni, Alfio. Numerical models for differential problems [on line]. Springer Verlag-Milano, 2009 [Consultation: 21/06/2023]. Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-88-470-1071-0. ISBN 9788847010710.
- Zienkiewicz, O. C. [et al.]. The Finite element method [on line]. 6th. Oxford: Butterworth Heinemann, [Consultation: 21/06/2023]. Available on: https://www-ingebook-com.recursos.biblioteca.upc.edu/ib/NPcd/IB_BooksVis?cod_primaria=1000187&codigo_libro=7854. ISBN 9780750664318.

Complementary:
- Donea, J.; Huerta, A. Finite element methods for flow problems [on line]. Chichester: John Wiley Sons, 2003 [Consultation: 21/06/2023]. Available on: https://onlinelibrary-wiley-com.recursos.biblioteca.upc.edu/doi/book/10.1002/0470013826. ISBN 0471496669.
- Elman, H.; Silvester, D.; Wathen, A. Finite elements and fast iterative solvers: with applications in incompressible fluid dynamics. Oxford University Press, 2005. ISBN 019852868 X.
- Ortega, James M. Numerical analysis: a second course. Philadelphia: Society for Industrial and Applied Mathematics, 1990. ISBN 0898712505.
- Ames, William F. Numerical methods for partial differential equations. Boston Academic Press, 1992. ISBN 012056761X.
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- Evans, G.; Blackledge, J.; Yardley, P. Numerical methods for partial differential equations. London: Springer Verlag, 1999. ISBN 354076125X.
- Johnson, Claes. Numerical solution of partial differential equations by the finite element method. Mineola, NY: Dover Publications, 2009. ISBN 9780486469003.
- Hughes, Thomas J. R. The Finite element method: linear static and dynamic finite element analysis. Mineola, New York: Dover Publications, 1987. ISBN 0486411818.