



# Course guide

## 200643 - MMIO - Models and Methods From Operations Research

**Last modified:** 26/06/2023

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 715 - EIO - Department of Statistics and Operations Research.  
**Degree:** MASTER'S DEGREE IN STATISTICS AND OPERATIONS RESEARCH (Syllabus 2013). (Compulsory subject).  
**Academic year:** 2023    **ECTS Credits:** 5.0    **Languages:** Spanish, English

### LECTURER

---

**Coordinating lecturer:** MARÍA PAZ LINARES HERREROS  
**Others:** Primer quadrimestre:  
DANIEL BAENA MIRABETE - A, B  
MARÍA PAZ LINARES HERREROS - A, B

### PRIOR SKILLS

---

Models and Methods of Operations Research is compulsory, students from Path 1 (graduate students in statistics or mathematics) will enroll Group A - Advanced, and students from Path 2 (all students from other degrees) will enroll Group B - Basic. Group A will be done in English, Group B in Spanish.

Group A follows the texts:

- Linear and nonlinear programming - Luenberger, D.G.; Ye, Y, Springer, 2016. ISBN: 9783319188416
- Numerical optimization - Nocedal, J.; Wright, S.J, Springer Science+Business Media, 2006. ISBN: 0387303030
- Integer programming - Wolsey, L.A, John Wiley & Sons, 1998. ISBN: 0471283665

Group B follows to a large extent the book

- Linear and nonlinear programming - Luenberger, D.G.; Ye, Y, Springer, 2016. ISBN: 9783319188416

### REQUIREMENTS

---

In order to follow properly this course and obtain its maximum output it is necessary to have previous basic knowledge on calculus with one and several variables, and to have basic knowledge of matrices and bases in vector spaces. It is highly recommended to know some basic programming techniques.

Group A has a higher level. In order to follow it properly and obtain its maximum output it is necessary to have basic knowledge of modeling techniques and methods in Operations Research and, specifically, in Linear Programming and Integer Linear.



## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

---

### Specific:

3. CE-2. Ability to master the proper terminology in a field that is necessary to apply statistical or operations research models and methods to solve real problems.
4. CE-3. Ability to formulate, analyze and validate models applicable to practical problems. Ability to select the method and / or statistical or operations research technique more appropriate to apply this model to the situation or problem.
5. CE-5. Ability to formulate and solve real problems of decision-making in different application areas being able to choose the statistical method and the optimization algorithm more suitable in every occasion.  
Translate to english
6. CE-6. Ability to use appropriate software to perform the necessary calculations in solving a problem.

### Transversal:

1. TEAMWORK: Being able to work in an interdisciplinary team, whether as a member or as a leader, with the aim of contributing to projects pragmatically and responsibly and making commitments in view of the resources that are available.
2. EFFECTIVE USE OF INFORMATION RESOURCES: Managing the acquisition, structuring, analysis and display of data and information in the chosen area of specialisation and critically assessing the results obtained.

## TEACHING METHODOLOGY

---

### Group A and B:

- Theoretical sessions: lectures in which the topics of the syllabus are introduced and discussed. The intranet will be used for making available teaching material related with the course: notes for some topics, resolved problems and previous exams.
- Problem-solving sessions: classes in which numerical problems concerning the subjects studied in the theory sessions are posed and solved. Students are given a certain amount of time to solve problems themselves, and then the problems will be resolved and discussed collectively.
- Lab: there will be lab sessions in order to introduce students to practical implementation and solution of Operations Research models using available software.

## LEARNING OBJECTIVES OF THE SUBJECT

---

The objectives of the course depend on the group.

Group A:

This course studies models and techniques of Operation Research, specifically Integer Programming. Special attention is given to the potential applications of the models. The application to classical combinatorial optimization models, like the traveling salesman problem, is also presented.

The main learning objectives of this course are:

- To provide a basic background in operations research, particularly in the field of Integer Programming.
- To familiarize students with methods for solving some practical applications of integer programming and combinatorial optimization problems.
- To know the possible modeling alternatives for the different types of optimization problems as well as their potential applications.
- To know the basic methodology of integer programming and, in particular, enumerative and cutting plane methods, as well as possible combinations of the above.
- To know results of duality theory and their implications in discrete programming.
- To know some basic heuristic methods for some combinatorial optimization problems.

Skills to achieve:

- The ability to find a suitable formulation and to design and implement a prototype method for the solution of a specific optimization problem.
- The ability to identify inequalities valid for typical problems in integer programming, such as the knapsack problem or the travelling salesman problem.
- The ability to formulate a Lagrangian relaxation for an optimization problem with constraints. The ability to determine the existence or not of a dual gap (or saddle points) for a particular optimization problem.

Group B:

It is an introductory course on of Operations Research models and methods. The main objective is to give an overall view of the main classes of models and their main potential applications, as well as of the techniques that must be used in each case. Basic versions will be studied of the most usual techniques linear and integer programming. Without ignoring the formal aspects, special attention will be given to the interpretation and application of the studied concepts.

The learning objectives of the course are:

- To provide a basic knowledge in the main models and techniques in Operations Research, as well as of the main applications. To familiarize students with basic methods that allow solving some practical applications.
- To know the possible modeling alternatives and the nature of the different classes of problems in Operations Research and their potential applications, with special emphasis in those related to statistical problems.
- To know the basic concepts and methodology of linear programming, duality and sensitivity analysis.
- To know the main Network Flow models, as well as their applications, including shortest paths and spanning trees.
- To know some basic concepts related to integer programming and, in particular, those related to cutting planes and basic enumerative methods.

Skills to achieve:

- The ability to formulate a suitable model for an specific mathematical optimization problem, and to implement it using a suitable modeling language.
- The ability to solve with the Simplex Algorithm small linear programming problems, and to answer simple sensitivity analysis questions.
- The ability to solve simple Network Flow models, including shortest paths and minimum spanning trees.
- The ability to apply basic integer programming techniques.

## STUDY LOAD

---

Type	Hours	Percentage
Hours large group	30,0	24.00
Hours small group	15,0	12.00
Self study	80,0	64.00

**Total learning time:** 125 h



## CONTENTS

### Topic 1: Introduction to models and formulations of Operations Research

**Description:**

Introduction to the course, highlighting the potential applications as well as the relevance in the discipline of models and mathematical optimization formulations.

**Full-or-part-time:** 17h

Theory classes: 5h

Laboratory classes: 2h

Self study : 10h

### Topic 3: Linear Programming methods and their properties

**Description:**

3.1 Bases and extreme points.

3.2 Basic concepts of duality and sensitivity analysis.

**Full-or-part-time:** 21h 20m

Theory classes: 5h

Laboratory classes: 3h

Self study : 13h 20m

### Topic 4: Network flow models: max flow, min-cost flow

**Description:**

4.1 Flow balance in a network.

4.2 Properties of linear formulations and their solutions.

4.3 Shortest path problems.

4.4 Spanning trees.

**Full-or-part-time:** 21h 20m

Theory classes: 5h

Laboratory classes: 3h

Self study : 13h 20m

### Topic 5: Basic models in integer programming and their properties

**Description:**

5.1 Cutting planes: Gomory cuts

5.2 Enumerative methods: branch-and-bound, branch-and-cut.

**Full-or-part-time:** 20h 20m

Theory classes: 5h

Laboratory classes: 2h

Self study : 13h 20m

## Topic 6: Advanced models and methods of Operations Research

### Description:

6.1 Combinatorial Optimization and its relation with integer Programming. Matching problems, packing, covering and partitioning. Facility location problems, vehicle routing and network design.

6.2 Exact solution methods.

i. Valid inequalities. The separation problem and cutting plane methods.

ii. Enumerative methods: implicit enumeration, branch-and-bound and branch-and-cut. Particular cases: Gomory cuts, Chvátal-Gomory, ...

6.3 Heuristic methods. Constructive methods (greedy, GRASP, ...), improving methods. Metaheuristics.

6.4 Lagrangean Relaxation in integer programming.

i. The Lagrangean Dual. Relation between dualization and convexification.

ii. The solution of the Lagrangean Dual: Non-smooth optimization, subgradient optimization.

6.5 Some combinatorial optimization problems.

i. The Knapsack Problem. Valid inequalities and facets: cover cuts. Separation and lifting.

ii. The Traveling Salesman Problem (TSP). Basic Properties and modeling alternatives. Valid inequalities and their separation: subtour elimination, 2-matching, comb inequalities.

**Full-or-part-time:** 75h

Theory classes: 40h

Practical classes: 20h

Laboratory classes: 15h

## GRADING SYSTEM

### GROUP A:

#### A.1. Continuous evaluation:

- There will be two partial exams, one for the first part of the course (P1) and one for the second (P2).
- There will be a final exam (EF) containing all the topics of the course, not compulsory, and which replaces both partial exams.
- Practical (PR): Completion of an assigned individual piece of work.

The final course result is calculated as follows:  $\max\{35\%P1+35\%P2+30\%PR, 70\%EF+30\%PR, 100\%EF\}$

#### A.2. Single act evaluation:

There will be a final exam (EF) covering all topics of the course.

### GROUP B:

#### B.1. Continuous Evaluation:

- E1: Partial exam about the first part of the course.
- E2: Exam about the second part of the course.
- ER: Individual exercise to be issued in date that will be announced.
- Class participation.

The final result will be:  $0.35 \cdot E1 + 0.35 \cdot E2 + 0.3 \cdot ER$ , where

E1: Grade of the partial exam about the first part of the course.

E2: Grade of the second part of the course.

ER: Grade of the practical exercise.

#### B.2. Single act evaluation:

There will be a final exam covering all topics of the course, and E1 and ER will be automatically waived.



## BIBLIOGRAPHY

---

### Basic:

- Luenberger, David G; Ye, Yinyu. Linear and nonlinear programming [on line]. 3rd ed. New York: Springer, cop. 2008 [Consultation: 04/07/2023]. Available on: <https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-0-387-74503-9>. ISBN 9780387745022.
- Wolsey, L. A. Integer programming. New York: John Wiley & Sons, 1998. ISBN 0471283665.

### Complementary:

- Padberg, M. Linear optimization and extensions. 2nd, revised and expanded ed. New York: Springer-Verlag, 1999. ISBN 3540658335.
- Fourer, Robert; Gay, David M; Kernighan, Brian W. AMPL : a modeling language for mathematical programming. 2nd ed. Pacific Grove, CA: Thomson/Brooks/Cole, cop. 2003. ISBN 0534388094.
- Cook, W. [et al.]. Combinatorial optimization. New York: Wiley, 1998. ISBN 047155894X.
- Bazaraa, M. S; Sherali, Hanif D; Shetty, C. M. Nonlinear programming : theory and algorithms. 3rd ed. Hoboken, N.J.: Wiley-Interscience, cop. 2006. ISBN 9780471486008.
- Bertsekas, Dimitri P. Nonlinear programming. 2nd ed. Belmont: Athena Scientific, cop. 1999. ISBN 1886529000.
- Ahuja, Ravindra K; Magnanti, Thomas L; Orlin, James B. Network flows : theory, algorithms, and applications. Englewood Cliffs, N.J.: Prentice Hall, cop. 1993. ISBN 013617549X.
- Nemhauser, G.L.; Wolsey, L.A. Integer and combinatorial optimization. New York: John Wiley and Sons, 1988. ISBN 047182819X.

## RESOURCES

---

### Computer material:

- CPLEX. Software for the solution of integer programming problems
- AMPL. Modeling language for mathematical optimization