UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Course guide
34952 - AG - Algebraic Geometry

Last modified: 19/05/2022

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2022 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: MARIA ALBERICH CARRAMIÑANA

Others: Segon quadrimestre:
MARIA ALBERICH CARRAMIÑANA - A
PEDRO PASCUAL GAINZA - A

PRIOR SKILLS

Aquaintance with mathematical computations, both by hand and with a computer, and mathematical reasoning, including proofs.

REQUIREMENTS

Basic abstract Algebra, Topology and Differential Geometry.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Roughly 50% of the class time will be devoted to the master classes, in which the lecturer will discuss the course topics. The other half of the class time will be structured as a problem class, in which the students will solve in the blackboard problems from a proposed list, based on the course syllabus, and their solutions will be discussed by the class.

LEARNING OBJECTIVES OF THE SUBJECT

The course consists of two distinct parts: global algebraic geometry and local algebraic geometry focusing on plane-curve singularities. The main objective of the first part of the course is to introduce the student to the Algebraic Geometry of affine and projective varieties, both algebraically over a field (Q, finite fields) and analytically over the real, and specially over the complex numbers.
The second part aims to give an insight into singularity theory of plane curves and a geometric theory of valuations of the ring of convergent series of two variables over the complex numbers.

The course will be based on many examples, stressing the geometric interest of the subject. The topic of the final lectures will depend on the interests of the audience, with a view towards the assigned final projects of the students.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Chapter 1: Algebraic varieties

Description:
Affine algebraic varieties. Nullstellensatz. Ring of regular functions. Subvarieties. Products of varieties, fibered products. Separation axiom.

Full-or-part-time: 13h
Theory classes: 6h
Self study : 7h

Chapter 2: Projective varieties

Description:
Projective Nullstellensatz and projective varieties. Elimination theory. Examples: grassmannians.

Full-or-part-time: 13h
Theory classes: 6h
Self study : 7h

Chapter 3: Maps and morphisms

Description:
Basic properties. Noether normalization theorem. Zariski's main theorem. Proper maps. Normalization. Resolution of singularities: blow-ups and Hironaka's theorem.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 4: Sheaves

Description:
Sheaves on a paracompact topological space, cohomology. Coherent sheaves on an algebraic variety: the canonical and hyperplane section sheaves, Riemann-Roch for curves. The Dolbeault complex over a complex analytic manifold: Hodge theory.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 5: Parametrizing branches of plane curves

Description:
Newton-Puiseux algorithm, Weierstrass preparation and division theorems, intersection multiplicity.

Full-or-part-time: 13h
Theory classes: 6h
Self study : 7h

Chapter 6: Infinitely near points

Description:
Proximity, Enriques diagrams, rings in successive neighbourhoods.

Full-or-part-time: 13h
Theory classes: 6h
Self study : 7h

Chapter 7: Topological and analytic classification of plane curves

Description:
Equisingularity, semigroup of values, Milnor and Tjurina numbers and other invariants.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 8: Valuations and complete ideals

Description:
Classification of valuations, Zariski decomposition of complete ideals.

Full-or-part-time: 18h
Theory classes: 8h
Self study : 10h

Chapter 9: Final projects

Description:
The final works of the subject on the topics chosen by the students will be presented by the students themselves and commented by the course lecturers.

Full-or-part-time: 24h
Theory classes: 4h
Self study : 20h

GRADING SYSTEM

Students who solve enough problems on the blackboard in the problem class pass the course. If they want to improve their grade from pass towards top score they will be assigned a final project, which will be to study and lecture on an additional topic at the end of the course.
Students who have not participated enough in the problem class, or still want to improve on their grade after problem class and additional lecture, will have to take a final exam of approximately 4 hours.

EXAMINATION RULES.

The problem list for participation in problem class will be published at the start of every course unit. Students will prepare these problems at home.
The topics for optional, grade increasing lectures at the end of the course will be proposed around Easter. Students will prepare these lectures at home.
Students who take the final exam will have to do so without any notes, books or material whatsoever.

BIBLIOGRAPHY

Basic:
- Harris, Joe. Algebraic geometry: a first course. New York: Springer Verlag, cop. 1992. ISBN 0387977163.
- Griffiths, Phillip ; Harris, Joseph. Principles of algebraic geometry. New York: Wiley, cop. 1978. ISBN 0471327921.
- Shafarevich, I.R. Basic algebraic geometry. 2nd. rev. and expanded ed. Berlin: Springer Verlag, 1994. ISBN 3540548122.
- Reid, Miles. Undergraduate algebraic geometry. Cambridge: Cambridge : Cambridge University Press, 1990. ISBN 0521356628.
- Smith, Karen E. An invitation to algebraic geometry. New York: Springer Verlag, 2000. ISBN 038798980.
- Casas Alvero, Eduardo. Singularities of plane curves. Cambridge: Cambridge University Press, 2000. ISBN 0521789591.

Complementary:
- Voisin, Claire. Hodge theory and complex algebraic geometry v1. Cambridge University Press, ISBN 0521802601.
- Beauville, Arnaud. Complex algebraic surfaces. 2nd ed. Cambridge University Press, 1996. ISBN 0521498422.
- Wall, C. T. C. Singular points of plane curves. Cambridge, UK ; New York: Cambridge University Press, cop. 2004. ISBN 0521547741.
- Chenciner, Alain. Courbes algébriques planes [on line]. Berlin: Springer, 2008Available on: https://link-springer-com.recursos.biblioteca.upc.edu/book/10.1007/978-3-540-33708-9. ISBN 9783540337072.