



## Course guides

# 34952 - AG - Algebraic Geometry

Last modified: 17/04/2021

**Unit in charge:** School of Mathematics and Statistics  
**Teaching unit:** 749 - MAT - Department of Mathematics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).  
(Optional subject).

**Academic year:** 2021    **ECTS Credits:** 7.5    **Languages:** English

### LECTURER

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**Coordinating lecturer:** PEDRO PASCUAL GAINZA

**Others:** Segon quadrimestre:  
PEDRO PASCUAL GAINZA - A

### PRIOR SKILLS

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Aquaintance with mathematical computations, both by hand and with a computer, and mathematical reasoning, including proofs.

### REQUIREMENTS

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Basic abstract Algebra, Topology and Differential Geometry.

### DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

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**Specific:**

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**

4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### TEACHING METHODOLOGY

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Roughly 50% of the class time will be devoted to the master classes, in which the lecturer will discuss the course topics. The other half of the class time will be structured as a problem class, in which the students will solve in the blackboard problems from a proposed list, based on the course syllabus, and their solutions will be discussed by the class.



## LEARNING OBJECTIVES OF THE SUBJECT

The main objective of the course is to introduce the student to the Algebraic Geometry of affine and projective varieties, both algebraically over a field ( $\mathbb{Q}$ , finite fields) and analytically over the real, and specially over the complex numbers. The course will be based on many examples, stressing the geometric interest of the subject. The topic of the final lectures will depend on the interests of the audience, with a view towards the assigned final projects of the students.

## STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

**Total learning time:** 187.5 h

## CONTENTS

### Chapter 1: Algebraic equations

**Description:**

Introduction: how systems of algebraic equations determine ideals in the ring of functions and, in the case of equations over the real or complex numbers, its solutions form manifolds with a given dimension and singularities in their closure.

**Full-or-part-time:** 15h

Theory classes: 6h

Self study : 9h

### Chapter 2: Algebraic varieties

**Description:**

Affine algebraic varieties. Nullstellensatz. Ring of regular functions. Subvarieties. Products of varieties, fibered products. Separation axiom.

**Full-or-part-time:** 13h

Theory classes: 6h

Self study : 7h

### Chapter 3: Projective varieties

**Full-or-part-time:** 9h

Theory classes: 4h

Self study : 5h

### Chapter 4: Maps and morphisms

**Description:**

Basic properties. Noether normalization theorem. Zariski's main theorem. Proper maps. Normalization. Resolution of singularities: blow-ups and Hironaka's theorem.

**Full-or-part-time:** 13h

Theory classes: 6h

Self study : 7h



### Chapter 5: Complex analytic varieties

**Description:**

Tangent spaces. Nonsingular points. Smooth maps. Global topology of varieties: fundamental class, degree of morphisms, intersection numbers. Applications: determinantal varieties, Grassmannians, parametrizing varieties...

**Full-or-part-time:** 14h

Theory classes: 8h

Self study : 6h

### Chapter 6: Sheaves

**Description:**

Sheaves on a paracompact topological space, cohomology. Coherent sheaves on an algebraic variety: the canonical and hyperplane section sheaves, Riemann-Roch for curves. The Dolbeault complex over a complex analytic manifold: Hodge theory.

**Full-or-part-time:** 18h

Theory classes: 8h

Self study : 10h

### Chapter 7: Final projects

**Description:**

The topics of the final projects made by course students, explained by themselves and by the course lecturer.

**Full-or-part-time:** 12h

Theory classes: 4h

Self study : 8h

## GRADING SYSTEM

Students who solve enough problems on the blackboard in the problem class pass the course. If they want to improve their grade from pass towards top score they will be assigned a final project, which will be to study and lecture on an additional topic at the end of the course.

Students who have not participated enough in the problem class, or still want to improve on their grade after problem class and additional lecture, will have to take a final exam of approximately 4 hours.

## EXAMINATION RULES.

The problem list for participation in problem class will be published at the start of every course unit. Students will prepare these problems at home.

The topics for optional, grade increasing lectures at the end of the course will be proposed around Easter. Students will prepare these lectures at home.

Students who take the final exam will have to do so without any notes, books or material whatsoever.



## BIBLIOGRAPHY

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### Basic:

- Harris, Joe. Algebraic geometry: a first course. New York: Springer Verlag, 1992. ISBN 0-387-97716-3.
- Griffiths, Phillip ; Harris, Joseph. Principles of algebraic geometry. John Wiley and Sons, 1978.
- Shafarevich, I.R. Basic algebraic geometry. 2nd. rev. and expanded ed. Berlin: Springer Verlag, 1994. ISBN 3540548122.
- Reid, Miles. Undergraduate algebraic geometry. Cambridge: Cambridge U.P., 1990. ISBN 0-521-35662-8.
- Smith, K. et al. An invitation to algebraic geometry. New York: Springer Verlag, 2000. ISBN 0-387-97716-3.

### Complementary:

- Voisin, Claire. Hodge theory and complex algebraic geometry 1. Cambridge U.P.,
- Beauville, A.. Complex algebraic surfaces. 2nd ed. Cambridge U.P., 1996.