Course guides
34953 - NT - Number Theory

Last modified: 17/04/2021

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2021 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: ANA RIO DOVAL

Others: Primer quadrimestre:
JOAN CARLES LARIO LOYO - A
ANA RIO DOVAL - A

PRIOR SKILLS

Basic knowledge of algebraic structures: groups, rings and fields.

REQUIREMENTS

Basic material covered in any standard courses on arithmetic, group theory and Galois theory. Although it is not strictly necessary, any background on algebraic curves, elliptic curves and basic number theory. But the course will be completely self-contained.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Most of the lectures will take place on the blackboard (replaced by online lessons if necessary), explaining carefully the contents of the course and providing as much explicit examples, exercises and applications as possible. The students will be encouraged to consult suitable references and to discuss between them and with the professor in order to achieve a good understanding of the material.

LEARNING OBJECTIVES OF THE SUBJECT

1) Algebraic number theory.
2) Number theory in function fields.
2) Cyclotomic theory.

STUDY LOAD

Type	Hours	Percentage
Hours large group	60,0	32.00
Self study	127,5	68.00

Total learning time: 187.5 h

CONTENTS

Algebraic Number Theory

Description:
Introduction
Cyclotomic extensions
Cyclotomic polynomials

Full-or-part-time: 10h
Theory classes: 10h

Number Theory in function fields

Description:
Functions fields over finite fields
Carlitz polynomials
Carlitz extensions

Full-or-part-time: 10h
Theory classes: 10h

Cyclotomic Theory

Description:
Cyclotomic integers
Cyclotomic units
Unique factorization
Class numbers
Galois action
Kronecker-Weber theorem
Regular polygons
Fermat equation
Quadratic reciprocity

Carlitz modules
Galois action
Carlitz-Hayes theorem
Cyclotomic and Carlitz analogies
Quadratic reciprocity
Drinfeld modules

Full-or-part-time: 40h
Theory classes: 40h

GRADING SYSTEM

There will be a final exam. Optionally the qualification might be obtained based on:
1) Active participation of the student during the course,
2) Resolution of exercises suggested in class and,
3) Ellaboration of a document in which the student develops in more detail and depth some of the material of the course.

EXAMINATION RULES.

Solved exercises and works must be delivered according to schedule.

BIBLIOGRAPHY

Basic:
- Marcus, D. A.. Number fields [on line]. 2a. Springer, 2018 [Consultation: 21/06/2021]. Available on: https://ebookcentral.proquest.com/lib/upcatalunya-ebooks/detail.action?docID=6315071. ISBN 978-3-319-90232-6.
- Washington, Lawrence C. Introduction to cyclotomic fields. 2a. Springer, 1997. ISBN 978-0-387-94762-4.
- Samuel, Pierre. Algebraic theory of numbers [on line]. Dover, 2008 [Consultation: 21/06/2021]. Available on: http://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db=nlabk&AN=1152495. ISBN 978-0486466668.
- Lang, Serge. Cyclotomic Fields I and II. 2a. Springer, 1990. ISBN 978-0-387-96671-7.
- Weiss, Edwin. Algebraic Number Theory [on line]. Dover, 2012 [Consultation: 21/06/2021]. Available on: http://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db=nlabk&AN=1151112. ISBN 9780486154367.
- Goss, David. Basic structures of function field arithmetic. Springer, 1998. ISBN 978-3-540-63541-3.
- Rosen, Michael. Number theory in function fields [on line]. Springer, 2002 [Consultation: 18/11/2020]. Available on: http://search.ebscohost.com/login.aspx?direct=true&scope=site&db=nlebk&db=nlabk&AN=2536816. ISBN 978-0387953359.

RESOURCES

Computer material:
- SAGE. Mathematical Software
- Matlab. Mathematical software