Course guides
34956 - DG - Discrete and Algorithmic Geometry

Last modified: 17/04/2021

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2021 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: JULIAN THORALF PFEIFLE

Others: Primer quadrimestre:
CLEMENS HUEMER - A
JULIAN THORALF PFEIFLE - A
RODRIGO IGNACIO SILVEIRA ISOBA - A

PRIOR SKILLS

- Elementary combinatorics.
- Elementary graph theory.
- Elementary algorithmics.
- Elementary data structures.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Theory classes will be used to present and develop the contents of the course. Most of the topics will be presented by the instructors, but there can be some sessions devoted to students presentations.

There will be lists of problems, which will not contain solutions. Problems will be designed to help students deepen and mature their command of the concepts and techniques presented in class. Some problems will be solved in class, some will be left as homework. In the problem sessions, the goal will be to propose and analyze alternative strategies to solve each problem, and to show how the results presented in class are applied. Most of the problems solved in class will be presented by the students.

LEARNING OBJECTIVES OF THE SUBJECT

Discrete, combinatorial and computational geometry are facets of a common body of knowledge that integrates fundamental elements from mathematics -mainly from algebra, topology and classical branches of geometry- with elements and problems from theoretical computer science and its applications.

The area focuses on the combinatorial and structural study of discrete geometric objects, as well as the design of algorithms to construct or analyze them. Among the objects studied, we can mention discrete sets of points, curves and manifolds, polytopes, convex bodies, packings, space decompositions, graphs, and geometric matroids.

By the end of the course, students should:
- Be able to recognize and formally express discrete geometric problems.
- Be able to discretize geometric problems, when possible.
- Be able to apply combinatorial techniques, as well as data structures and algorithms to discrete geometric problems.
- Be able to search the bibliography, and to understand the scientific literature on the subject.
- Be aware of the wide range of fields and problems to which discrete geometry results apply.
- Be aware of the most commonly used software in the field.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Preliminaries

Description:
Computational complexity. Data structures. Representation of geometric objects.

Full-or-part-time: 12h 30m
Theory classes: 4h
Self study : 8h 30m

Convexity

Description:
Convex hull computation. Linear programming in low dimensions.

Full-or-part-time: 19h
Theory classes: 4h
Laboratory classes: 2h
Self study : 13h

Decompositions and arrangements

Description:
Subdivisions and triangulations of point sets and polygons. Visibility and motion planning. Duality. Special decompositions in dimension 2. The zone theorem. Incremental construction and randomized algorithms. Complexity. Levels and k-sets.

Full-or-part-time: 31h
Theory classes: 7h
Laboratory classes: 3h
Self study : 21h

Proximity Structures

Description:
Proximity problems. Voronoi diagram, Delaunay triangulation. Shape reconstruction.

Full-or-part-time: 31h
Theory classes: 7h
Laboratory classes: 3h
Self study : 21h

Polytopes and Subdivisions of Point Sets

Description:
Homogeneous coordinates. Polytopes: faces and boundary structure; examples; operations on
polytopes (polarity, products, etc.). Point sets: subdivisions and triangulations (including Delaunay and Voronoi).

Full-or-part-time: 38h
Theory classes: 10h
Laboratory classes: 3h
Self study : 25h

Topological Data Analysis

Description:
Homology: Simplicial homology, cell complexes.
Persistent homology: persistent Betti numbers, barcodes
Discrete Morse Theory: simplicial collapses, acyclic matchings, critical cells
Applications

Full-or-part-time: 48h
Theory classes: 12h
Laboratory classes: 4h
Self study : 32h

Software

Description:
Polymake, Curved Spaces, etc.

Full-or-part-time: 9h
Laboratory classes: 2h
Self study : 7h

GRADING SYSTEM

The course consists in two parts, each contributes with 50 % to the final grade.
For each part: Students will obtain marks by turning in their solutions to problems from the problem sets (50%), by presenting solutions to problems or a research paper (15 %), and there will be an exam (35 %).

BIBLIOGRAPHY

Basic:
- Boissonnat, J. D.; Yvinec, M. Algorithmic geometry. Cambridge: Cambridge University Press, 1997. ISBN 0521565294.
- Conway, John Horton; Sloane, N. J. A. Sphere packings, lattices and groups. 3rd ed. Berlin: Springer, 1999. ISBN 0387985859.
- Edelsbrunner, Herbert. Algorithms in combinatorial geometry. Berlín: Springer, 1987. ISBN 354013722X.
- Matousek, Jirí. Lectures on discrete geometry. New York: Springer, 2002. ISBN 0387953736.
- Pach, János; Agarwal, Pankaj K. Combinatorial geometry. New York: John Wiley & Sons, 1995. ISBN 0471588903.
- Ziegler, Günter M. Lectures on polytopes. New York: Springer-Verlag, 1995. ISBN 038794365X.
- Berg, Mark de; Cheong, Otfried; Kreveld, Marc van; Overmars, Mark. Computational geometry: algorithms and applications [on line]. 3rd ed. revised. Berlin: Springer, 2008 [Consultation: 18/11/2020]. Available on: https://ebookcentral.proquest.com/lib/upcatalunya-ebooks/detail.action?docID=6311912. ISBN 9783540779735.
- Beck, Matthias ; Robins, Sinai. Computing the continuous discretely : integer-point enumeration in polyhedra [on line]. New York: Springer, 2007 [Consultation: 20/05/2020]. Available on: http://dx.doi.org/10.1007/978-0-387-46112-0. ISBN 9780387291390.

Complementary:
- Bokowski, Jürgen. Computational oriented matroids : equivalence classes of matrices within a natural framework. Cambridge: Cambridge University Press, 2006. ISBN 9780521849302.
- Schurmann, Achill. Computational geometry of positive definite quadratic forms : polyhedral reduction theories, algorithms, and applications. Providence: AMS ULECT-48, 2009. ISBN 9780821847350.
- Weeks, Jeffrey R. The shape of space. 2nd. ed. New York: M. Dekker, 2002. ISBN 0824707095.
- Richter-Gebert, Jürgen. Perspectives on projective geometry: a guided tour through real and complex geometry [on line]. Berlin: Springer, 2011 [Consultation: 19/05/2020]. Available on: https://doi.org/10.1007/978-3-642-17286-1. ISBN 9783642172854.

RESOURCES

Audiovisual material:
- Mathfilm festival 2008 [Enregistrament vídeo]: a collection of mathematical videos. Berlin : Springer, 2008. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003546329706711&context=L&vid=34CSUC_UPC:VU1&lang=ca
- Videomath Festival at International Congress of Mathematicians, Berlin, Germany 1998 [Enregistrament vídeo] / edited and produced Hans Christian Hege, Konrad Polthier. [Berlin] : Springer, 1998. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001865709706711&context=L&vid=34CSUC_UPC:VU1&lang=ca
- Not knot [Enregistrament vídeo] / directed by Charlie Gunn and Delle Maxwell ; [written by David Epstein ... [et al.]]. Minnesota : Geometry Center, University of Minnesota, 1991. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991001445799706711&context=L&vid=34CSUC_UPC:VU1&lang=ca
- Flatland [Enregistrament vídeo] : a journey of many dimensions / written by Seth Caplan, Dano Johnson, Jeffrey Travis ; directed by Jeffrey Travis, Dano Johnson. [S.l.] : Flat World Productions, cop. 2007. https://discovery.upc.edu/discovery/fulldisplay?docid=alma991003683649706711&context=L&vid=34CSUC_UPC:VU1&lang=ca