Course guide
34956 - DG - Discrete and Algorithmic Geometry

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).
(Optional subject).
Academic year: 2022  ECTS Credits: 7.5  Languages: English

LECTURER
Coordinating lecturer: CLEMENS HUEMER
Others: Primer quadrimestre:
CLEMENS HUEMER - A
JULIAN THORALF PFEIFLE - A
RODRIGO IGNACIO SILVEIRA ISOBA - A

PRIOR SKILLS
- Elementary combinatorics.
- Elementary graph theory.
- Elementary algorithmics.
- Elementary data structures.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
TEACHING METHODOLOGY

Theory classes will be used to present and develop the contents of the course. Most of the topics will be presented by the instructors, but there can be some sessions devoted to students presentations.

There will be lists of problems, which will not contain solutions. Problems will be designed to help students deepen and mature their command of the concepts and techniques presented in class. Some problems will be solved in class, some will be left as homework. In the problem sessions, the goal will be to propose and analyze alternative strategies to solve each problem, and to show how the results presented in class are applied. Most of the problems solved in class will be presented by the students.

LEARNING OBJECTIVES OF THE SUBJECT

Discrete, combinatorial and computational geometry are facets of a common body of knowledge that integrates fundamental elements from mathematics -mainly from algebra, topology and classical branches of geometry- with elements and problems from theoretical computer science and its applications.

The area focuses on the combinatorial and structural study of discrete geometric objects, as well as the design of algorithms to construct or analyze them. Among the objects studied, we can mention discrete sets of points, curves and manifolds, polytopes, convex bodies, packings, space decompositions, graphs, and geometric matroids.

By the end of the course, students should:
- Be able to recognize and formally express discrete geometric problems.
- Be able to discretize geometric problems, when possible.
- Be able to apply combinatorial techniques, as well as data structures and algorithms to discrete geometric problems.
- Be able to search the bibliography, and to understand the scientific literature on the subject.
- Be aware of the wide range of fields and problems to which discrete geometry results apply.
- Be aware of the most commonly used software in the field.

STUDY LOAD

<table>
<thead>
<tr>
<th>Type</th>
<th>Hours</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self study</td>
<td>127,5</td>
<td>68.00</td>
</tr>
<tr>
<td>Hours large group</td>
<td>60,0</td>
<td>32.00</td>
</tr>
</tbody>
</table>

Total learning time: 187.5 h

CONTENTS

Preliminaries

Description:
Computational complexity. Data structures. Representation of geometric objects.

Full-or-part-time: 12h 30m
Theory classes: 4h
Self study: 8h 30m
### Convexity

**Description:**
Convex hull computation. Linear programming in low dimensions.

**Full-or-part-time:** 19h
- Theory classes: 4h
- Laboratory classes: 2h
- Self study: 13h

### Decompositions and arrangements

**Description:**

**Full-or-part-time:** 31h
- Theory classes: 7h
- Laboratory classes: 3h
- Self study: 21h

### Proximity Structures

**Description:**
Proximity problems. Voronoi diagram, Delaunay triangulation. Shape reconstruction.

**Full-or-part-time:** 31h
- Theory classes: 7h
- Laboratory classes: 3h
- Self study: 21h

### Polytopes and Subdivisions of Point Sets

**Description:**
Homogeneous coordinates. Polytopes: faces and boundary structure; examples; operations on polytopes (polarity, products, etc.). Point sets: subdivisions and triangulations (including Delaunay and Voronoi).

**Full-or-part-time:** 38h
- Theory classes: 10h
- Laboratory classes: 3h
- Self study: 25h

### Topological Data Analysis

**Description:**
Homology: Simplicial homology, cell complexes.
Persistent homology: persistent Betti numbers, barcodes
Discrete Morse Theory: simplicial collapses, acyclic matchings, critical cells
Applications

**Full-or-part-time:** 48h
- Theory classes: 12h
- Laboratory classes: 4h
- Self study: 32h
Software

Description:
Polymake, Curved Spaces, etc.

Full-or-part-time: 9h
Laboratory classes: 2h
Self study: 7h

GRADING SYSTEM

The course consists in two parts, each contributes with 50 % to the final grade.
For each part: Students will obtain marks by turning in their solutions to problems from the problem sets (50%), by presenting solutions to problems or a research paper (15 %), and there will be an exam (35 %).

BIBLIOGRAPHY

Basic:

Complementary:
RESOURCES

Audiovisual material: