34956 - DG - Discrete and Algorithmic Geometry

Coordinating unit: 200 - FME - School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics
Academic year: 2018
Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional)
ECTS credits: 7,5 Teaching languages: English

Teaching staff

Coordinator: CLEMENS HUEMER
Others: Primer quadriandre:
        CLEMENS HUEMER - A
        JULIAN THORALF PFEIFLE - A

Prior skills

- Elementary combinatorics.
- Elementary graph theory.
- Elementary algorithmics.
- Elementary data structures.

Degree competences to which the subject contributes

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
3. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
4. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
5. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
6. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
7. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
8. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.
Discrete, combinatorial and computational geometry are facets of a common body of knowledge that integrates fundamental elements from mathematics -mainly from algebra, topology and classical branches of geometry- with elements and problems from theoretical computer science and its applications.

The area focuses on the combinatorial and structural study of discrete geometric objects, as well as the design of algorithms to construct or analyze them. Among the objects studied, we can mention discrete sets of points, curves and manifolds, polytopes, convex bodies, packings, space decompositions, graphs, and geometric matroids.

By the end of the course, students should:
- Be able to recognize and formally express discrete geometric problems.
- Be able to discretize geometric problems, when possible.
- Be able to apply combinatorial techniques, as well as data structures and algorithms to discrete geometric problems.
- Be able to search the bibliography, and to understand the scientific literature on the subject.
- Be aware of the wide range of fields and problems to which discrete geometry results apply.
- Be aware of the most commonly used software in the field.

### Study load

<table>
<thead>
<tr>
<th><strong>Total learning time:</strong> 187h 30m</th>
<th>Hours large group: 60h</th>
<th>32.00%</th>
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<tbody>
<tr>
<td><strong>Self study:</strong> 127h 30m</td>
<td>68.00%</td>
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## Content

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<tr>
<th>Preliminaries</th>
<th>Learning time: 12h 30m</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 4h</td>
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<td>Self study : 8h 30m</td>
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**Description:**
Computational complexity. Data structures. Representation of geometric objects.

<table>
<thead>
<tr>
<th>Convexity</th>
<th>Learning time: 19h</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 4h</td>
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<tr>
<td></td>
<td>Laboratory classes: 2h</td>
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<td>Self study : 13h</td>
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**Description:**
Convex hull computation. Linear programming in low dimensions.

<table>
<thead>
<tr>
<th>Decompositions and arrangements</th>
<th>Learning time: 31h</th>
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<tbody>
<tr>
<td></td>
<td>Theory classes: 7h</td>
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<tr>
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<td>Laboratory classes: 3h</td>
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<td>Self study : 21h</td>
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**Description:**

<table>
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<tr>
<th>Proximity Structures</th>
<th>Learning time: 31h</th>
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<td>Theory classes: 7h</td>
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<tr>
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<td>Laboratory classes: 3h</td>
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<td>Self study : 21h</td>
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**Description:**
Proximity problems. Voronoi diagram, Delaunay triangulation. Shape reconstruction.
### Polytopes and Subdivisions of Point Sets

**Description:**
Homogeneous coordinates. Polytopes: faces and boundary structure; examples; operations on polytopes (polarity, products, etc.). Point sets: subdivisions and triangulations (including Delaunay and Voronoi).

**Learning time:** 38h  
Theory classes: 10h  
Laboratory classes: 3h  
Self study: 25h

### Lattice Geometry

**Description:**
Examples of lattices. Ehrhart's Theorem on integer points in polytopes. Brion's Theorem.

**Learning time:** 24h  
Theory classes: 6h  
Laboratory classes: 2h  
Self study: 16h

### Symmetry

**Description:**
Orbifolds and the Magic Theorem on symmetry groups in the plane. Exploitation of symmetry in linear optimization.

**Learning time:** 23h  
Theory classes: 6h  
Practical classes: 1h  
Self study: 16h

### Software

**Description:**
Polymake, Curved Spaces, etc.

**Learning time:** 9h  
Laboratory classes: 2h  
Self study: 7h
In general, there will be two or more exams during class hours, to be announced in advance. If so announced, students will also obtain marks by turning in their solutions to problems from the problem sets, and possibly presenting them at the blackboard.

In the case of a very small group, some exams may be replaced by personal work.

The exams and marks for the turned-in work will combine for the final qualification.
Bibliography

Basic:


Complementary:


Others resources:

Audiovisual material


Not knot [Enregistrament vídeo] / directed by Charlie Gunn and Delle Maxwell; [written by David Epstein ... [et al.]]. Minnesota: Geometry Center, University of Minnesota, 1991

Flatland [Enregistrament vídeo]: a journey of many dimensions / written by Seth Caplan, Dano Johnson, Jeffrey Travis; directed by Jeffrey Travis, Dano Johnson. [S.l.]: Flat World Productions, cop. 2007