Course guide
34957 - GT - Graph Theory

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.
Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010).
(Optional subject).

Academic year: 2022  ECTS Credits: 7.5  Languages: English

LECTURER

Coordinating lecturer: GUILLEM PERARNAU LLOBET
Others: Primer quadrimestre:
GUILLEM PERARNAU LLOBET - A
ORIOL SERRA ALBO - A

PRIOR SKILLS

Elementary Calculus and Linear Algebra; basic notions and skills in combinatorics and probability.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Combination of theoretical lectures and exercise classes, with student presenting their solutions to the proposed problems. The active participation of students is a requirement for the course assessment.
LEARNING OBJECTIVES OF THE SUBJECT

Basics of Graph Theory.
Graphs on surfaces and minors.
Random graphs.
Applications of random graphs: the probabilistic method.
Spectral techniques to the study of graphs.
Applications of spectral techniques: expansion and random walks.

STUDY LOAD

<table>
<thead>
<tr>
<th>Type</th>
<th>Hours</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Self study</td>
<td>127,5</td>
<td>68.00</td>
</tr>
<tr>
<td>Hours large group</td>
<td>60,0</td>
<td>32.00</td>
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Total learning time: 187.5 h

CONTENTS

Spectral graph theory

Description:
Introduction to spectral graph theory, graph expanders and applications to random walks on graphs.

Specific objectives:

Related competencies:
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
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MAMME-CE4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.
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Full-or-part-time: 16h
Theory classes: 16h
Minors and treewidth

Description:
Introduction to structural graph theory.

Specific objectives:

Related competencies:
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
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Full-or-part-time: 8h
Theory classes: 8h
Graphs on surfaces

**Description:**
Introduction to planar graphs and graphs embedded in other surfaces.

**Specific objectives:**

**Related competencies:**
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
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MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
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**Full-or-part-time:** 4h
Theory classes: 4h
Random graphs

Description:
Introduction to classical model of random graphs and its main combinatorial properties.

Specific objectives:
Erdos-Rényi model of random graphs.
Properties of almost all graphs.
First and second moment methods.
The probabilistic method.
Threshold functions.
Method of Moments
Chernoff's inequality
Component phase transitions

Related competencies:
MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
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Full-or-part-time: 12h
Theory classes: 12h
An introduction to Graph Theory

Description:
Introduction to random graphs, main properties and classical theorems.

Specific objectives:
- Basic terminology and notation
- Paths and cycles
- Distance and Diameter
- Connectivity
- Trees
- Matchings
- Colorings

Related competencies:
- MAMME-CE2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
- MAMME-CE1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
- MAMME-CE3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
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Full-or-part-time: 12h
Theory classes: 12h

GRADING SYSTEM

The assessment of the course is as follows:
- weekly work on the proposed problems and their presentation during the lectures, 30% of the mark
- a final comprehensive exam on the course topics, 70% of the mark

EXAMINATION RULES.

The active participation in the course is a requirement for the final assessment.
BIBLIOGRAPHY

Basic:
- Alon, Noga; Spencer Joel. The Probabilistic Method. 2016. Wiley,

Complementary: