# Course guides
# 34959 - CM - Computational Mechanics

**Last modified:** 28/05/2021

**Unit in charge:** School of Mathematics and Statistics
**Teaching unit:** 749 - MAT - Department of Mathematics.
751 - DECA - Department of Civil and Environmental Engineering.
748 - FIS - Department of Physics.

**Degree:** MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

**Academic year:** 2021 **ECTS Credits:** 7.5 **Languages:** English

## LECTURER

**Coordinating lecturer:** JOSE JAVIER MUÑOZ ROMERO

**Others:** Segon quadrimestre:
ALVARO MESEGUER SERRANO - A
JOSE JAVIER MUÑOZ ROMERO - A
ANTONIO RODRIGUEZ FERRAN - A

## PRIOR SKILLS

Basic knowledge of numerical methods
Basic knowledge of partial differential equations

## DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

**Specific:**
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

**Transversal:**
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

## TEACHING METHODOLOGY

Four elements will be combined:
-Theory classes, where the main concepts will be presented.
-Practical classes with Matlab code in the computer room, with emphasis on the computational aspects.
-Lists of short assignments.
-Course projects in groups to be presented orally at the end of the course.

Students will work on the assignments and course projects individually or in groups.

## LEARNING OBJECTIVES OF THE SUBJECT

The main objective is to provide a general perspective of the broad field of computational mechanics, covering both the modelling and the computational aspects. A broad range of problems is addressed: solids, fluids and fluid-solid interaction; linear and nonlinear models; static and dynamic problems. Some emphasis is put on applications in biomechanical problems. By the end of the course, the students should:
- Be able to choose the appropriate type of model for a specific simulation
- Be familiar with the mathematical objects (tensors) and differential operators used in computational mechanics
- Be aware of the different level of complexity of various problems (e.g. linear vs. nonlinear,
static vs. dynamic).

## STUDY LOAD

| Type | Hours | Percentage |
|---|---|---|
| Self study | 127,5 | 68.00 |
| Hours large group | 60,0 | 32.00 |

**Total learning time:** 187.5 h

## CONTENTS

### CONTINUUM MECHANICS

**Description:**
Motivation. Definition of continuous media. Equation of motion: Eulerian and Lagrangian
descriptions. Time derivatives. Strains: deformation gradient, Green and Euler-Almansi tensors;
elongation and shear; small strains. Stresses: body and surface forces; Cauchy stress tensor.
Balance equations: Reynolds transport theorem; mass balance; momentum balance. Constitutive equations. Applications.

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

### COMPUTATIONAL ELASTICITY

**Description:**
Basic concepts and motivation. Elastic constitutive equation. Displacement formulation: Navier
equations. Two-dimensional elasticity: plane stresses, plane strains and axisymmetry. Weak form
of the elastic problem. Finite element discretisation. Computational aspects. Applications in engineering and biomechanics.

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

## COMPUTATIONAL DYNAMICS

**Description:**
Weak form. Dynamic equation. Space discretisation (finite elements) and time discretisation. Solution methods: generalised eigen value problem and direct time integration. Euler, centred differences, HHT and Newmark methods. Stability, consistency and accuracy of numerical techniques in elastodynamics. Applications and realtion with reduced order techniques (Proper Orthogonal Decomposition).

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

## COMPUTATIONAL PLASTICITY, FRACTURE AND VISCOELASTICITY

**Description:**
Basic concepts and motivation. Non-linear problems. Geometrical and material non-linearity. One-dimensional plasticity: elastic and plastic strains;
elastoplastic constitutive equation; hardening. Multi-dimensional plasticity: stress and strain invariants; yield surface; plastic flow. Numerical time-integration of the constitutive equation: elastic prediction and plastic correction; iterative methods for the plastic correction. Applications. Phase field formulation and Finite Element discretisation.
Viscoelastic materials. Maxwell and Kelvin-Voigt one-dimensional models. Numerical solution. Extension to multiple dimensions.

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

## COMPUTATIONAL FLUID DYNAMICS

**Description:**
Basic concepts and motivation. Rate-of-deformation and spin tensors. Constitutive equation for Newtonian fluids. Euler equations for inviscid flow. Navier-Stokes equations for viscous flow in strong form and in weak form. Reynolds number. Stokes flow and potential flow. Finite element discretisation and numerical solution. Applications.

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

## COMPUTATIONAL METHODS FOR WAVE PROBLEMS

**Description:**
Basic concepts and motivation.
Acoustics: the wave equation. The scalar Helmholtz equation. Vibroacoustics: acoustic fluid-elastic solid interaction.
Computational aspects. Applications.
Electromagnetism: the Maxwell equations. Electrodynamics. The vectorial Helmholtz equation.
Computational aspects. Applications.

**Full-or-part-time:** 31h 15m
Theory classes: 8h
Practical classes: 2h
Self study : 21h 15m

## GRADING SYSTEM

Final exam (40%), assignment problems (30%), and course project (30%, evaluated with an oral presentation and a written report).

## BIBLIOGRAPHY

**Basic:**
- Clough, Ray W.; Penzien, J. Dynamics of structures. 2nd ed. New York: McGraw-Hill, 1993. ISBN 0071132414.
- Donea, Jean M.; Huerta, A. Finite element methods for flow problems. Chichester: John Wiley & Sons, 2003. ISBN 0471496669.
- Ihlenburg, F. Finite element analysis of acoustic scattering [on line]. New York: Springer-Verlag, 1998Available on: http://link.springer.com/book/10.1007%2Fb98828. ISBN 0387983198.
- Mase, G. Thomas; Mase, George E. Continuum mechanics for engineers. 3rd ed. Boca Raton: CRC, 2010.

**Complementary:**
- Holzapfel, Gerhard A. Nonlinear solid mechanics : a continuum approach for engineering. Chichester: John Wiley & Sons, cop. 2000. ISBN 978-0-471-82319-3.
- Taber, Larry A. Nonlinear theory of elasticity. Applications in Biomechanics. 2008. Singapore: World Scientific Publishing, 2004. ISBN 9812387358.
- Bonet, Javier; Wood, R. D. Nonlinear continuum mechanics for finite element analysis. 2nd ed. Cambridge: Cambridge University Press, 2008. ISBN 9780521838702.
- Marsden, Jerrold E.; Hugues, Thomas J. R. Mathematical foundations of elasticity. New York: Dover, 1994. ISBN 0486678652.
- Simo, J. C.; Hughes, T. J. R. Computational inelasticity [on line]. New York: Springer-Verlag, 1998 [Consultation: 19/05/2020]. Available on: http://link.springer.com/book/10.1007/b98904.
- Zienkiewicz O. C.;Taylor, R. L. The finite element method [on line]. 6th ed. Oxford: Butterworth Heinemann, 2005 [Consultation: 19/05/2020]. Available on: http://www.sciencedirect.com/science/book/9780750664318.
- Bathe, Klaus-Jürgen. Finite element procedures. New Jersey: Prentice-Hall, 1996. ISBN 0133014584.