Course guide
34962 - HS - Hamiltonian Systems

Last modified: 25/05/2022

Unit in charge: School of Mathematics and Statistics
Teaching unit: 749 - MAT - Department of Mathematics.

Degree: MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Optional subject).

Academic year: 2022 **ECTS Credits:** 7.5 **Languages:** English

LECTURER

Coordinating lecturer: PAU MARTIN DE LA TORRE

Others: Segon quadrimestre:
AMADEU DELSHAMS I VALDES - A
PAU MARTIN DE LA TORRE - A

PRIOR SKILLS

Knowledge of calculus, algebra and ordinary differential equations.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Specific:
1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:
5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

TEACHING METHODOLOGY

Standard exposition in front of the blackboard, resolution of exercices, completion of a project and attendance to the JISD summer school http://www.ma1.upc.edu/recerca/jisd

LEARNING OBJECTIVES OF THE SUBJECT

To comprehend the basic foundations of the theory of Hamiltonian systems, and to understand its applications to Celestial Mechanics and other fields.

STUDY LOAD

Type	Hours	Percentage
Self study	127,5	68.00
Hours large group	60,0	32.00

Total learning time: 187.5 h

CONTENTS

Hamiltonian formalism

Description:
Hamiltonian dynamical systems: symplectic maps, symplectic manifolds. Linear Hamiltonian systems and their application to the study of stability of equilibrium points. Canonical transformations.

Full-or-part-time: 28h
Theory classes: 10h
Self study : 18h

Celestial mechanics

Description:
The two body problem, first integrals. Resolution. The three body problem, different coordinates. The restricted three body problem. Central configurations. Periodic orbits, invariant manifolds.

Full-or-part-time: 34h
Theory classes: 12h
Self study : 22h

Geometric theory and invariant objects of Hamiltonian systems

Description:
Continuous and discrete dynamical systems, Poincaré map. Flow box Theorem. Noether Theorem. Periodic orbits. Continuation of periodic orbits. Lyapunov Center Theorem.

Full-or-part-time: 24h
Theory classes: 8h
Self study : 16h

Integrable systems

Description:
Complete integrability and Liouville-Arnold theorem. Action-Angle coordinates. Quasi-periodic flows on a torus, resonances.

Full-or-part-time: 10h
Theory classes: 4h
Self study : 6h

Quasi-integrable Hamiltonian systems

Description:
Examples of quasi-integrable systems. Small divisors and Diophantine inequalities. Averaging Theory. Lie Method. KAM Theory (Kolmogorov-Arnold Moser). Effective stability and Nekhoroshev theorem. Melnikov Potential. Arnold diffusion.

Full-or-part-time: 26h
Theory classes: 8h
Self study : 18h

Lagrangian systems and variational methods

Description:
Lagrangian systems. Legendre transformation. Principle of minimal action. Twist maps. Existence of periodic orbits. Aubry-Mather Theory.

Full-or-part-time: 12h
Theory classes: 4h
Self study : 8h

Hamiltonian Partial Differential Equations

Description:
Linear Hamiltonian Partial Differential Equations. Examples. Periodic, quasi-periodic and almost-periodic solutions. Nonlinear Hamiltonian Partial Differential Equations. Lyapunov stability/instability of invariant objects. Transfer of energy.

Full-or-part-time: 4h
Theory classes: 2h
Self study : 2h

- Interactions between Dynamical Systems and Partial Differential Equations

Description:
Summer School and Research workshop on topics between Dynamical Systems and Partial Differential Equations

Full-or-part-time: 49h 30m
Theory classes: 12h
Self study : 37h 30m

ACTIVITIES

JISD summer school

Description:
Attendance to the JISD summer school

Specific objectives:
To learn from oustanding researchers a view of the state of the art in several research topics, interacting with students of the rest of Spain and of the World.

GRADING SYSTEM

The students have to do some problems (60%) and a research work (25%). There will be also a final exam covering on the theoretical part of the subject (15%). Moreover, they will attend the JISD.

BIBLIOGRAPHY

Basic:

- Kanuf, Andreas. Mathematical physics: Classical mechanics. 1. Springer-Verlag, 2018. ISBN 978-3-662-55772-3.
- Marsden, Jerrold E; Ratiu, Tudor S. Introduction to mechanics and symmetry : a basic exposition of classical mechanical systems. 2a ed. New York [etc.]: Springer, 1999. ISBN 978-0-387-98643-2.
- Arnol'd, V. I.; Kozlov, Valerii V.; Neishtadt, Anatoly I. Mathematical aspects of classical and celestial mechanics [on line]. 3rd ed. Berlin: Springer-Verlag, 2006 [Consultation: 19/05/2020]. Available on: http://dx.doi.org/10.1007/978-3-540-48926-9. ISBN 3540282467.
- Katok, Anatole; Hasselblatt, Boris. Introduction to the modern theory of dynamical systems. Cambridge [etc.]: Cambridge University Press, 1997. ISBN 9780521575577.
- Celletti, Alessandra. Stability and chaos in celestial mechanics [on line]. Springer-Praxis, 2010 [Consultation: 19/05/2020]. Available on: https://ebookcentral.proquest.com/lib/upcatalunya-ebooks/detail.action?docID=993277. ISBN 978-3-540-85145-5.
- Treschev, Dmitry; Zubelevich, Oleg. Introduction to the perturbation theory of Hamiltonian systems [on line]. Berlin: Springer Verlag, 2010 [Consultation: 19/05/2020]. Available on: http://dx.doi.org/10.1007/978-3-642-03028-4. ISBN 978-3-642-03027-7.
- Meyer, Kenneth R.; Hall, Glen R.; Offin, Dan. Introduction to Hamiltonian dynamical systems and the n-body problem [on line]. 2nd ed. New York: Springer-Verlag, 2009 [Consultation: 19/05/2020]. Available on: http://dx.doi.org/10.1007/978-0-387-09724-4. ISBN 978-0-387-09723-7.
- Berti, Massimiliano. Nonlinear Oscillations of Hamiltonian PDEs [on line]. Boston, MA: Birkhäuser Boston, Inc., 2007 [Consultation: 19/05/2020]. Available on: http://dx.doi.org/10.1007/978-0-8176-4681-3. ISBN 978-0-8176-4680-6.
- Wintner, Aurel. The analytical foundations of celestial mechanics. Dover Publications, ISBN 978-0486780603.

RESOURCES

Hyperlink:

- Grup de sistemes dinàmicshttps://recerca.upc.edu/sd. Pàgina web del Grup de Sistemes Dinàmics de la UPC on es descriuen diversos projectes i els investigadors que hi treballen així com diverses activitats relacionades