

## 34965 - NMPDE - Numerical Methods for Partial Differential Equations

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| Coordinating unit: | 200 - FME - School of Mathematics and Statistics   |                     |         |
| Teaching unit:     | 749 - MAT - Department of Mathematics<br>751 - DECA - Department of Civil and Environmental Engineering        |                     |         |
| Academic year:     | 2017   |                     |         |
| Degree:            | MASTER'S DEGREE IN ADVANCED MATHEMATICS AND MATHEMATICAL ENGINEERING (Syllabus 2010). (Teaching unit Optional) |                     |         |
| ECTS credits:      | 7,5  | Teaching languages: | English |

### Teaching staff

|              |   |
|--------------|---|
| Coordinator: | SONIA FERNANDEZ MENDEZ  |
| Others:      | Primer quadrimestre:<br>SONIA FERNANDEZ MENDEZ - A<br>ABEL GARGALLO PEIRO - A |

### Prior skills

Basics on numerical methods, differential equations and calculus.

### Degree competences to which the subject contributes

Specific:

1. RESEARCH. Read and understand advanced mathematical papers. Use mathematical research techniques to produce and transmit new results.
2. MODELLING. Formulate, analyse and validate mathematical models of practical problems by using the appropriate mathematical tools.
3. CALCULUS. Obtain (exact or approximate) solutions for these models with the available resources, including computational means.
4. CRITICAL ASSESSMENT. Discuss the validity, scope and relevance of these solutions; present results and defend conclusions.

Transversal:

5. SELF-DIRECTED LEARNING. Detecting gaps in one's knowledge and overcoming them through critical self-appraisal. Choosing the best path for broadening one's knowledge.
6. EFFICIENT ORAL AND WRITTEN COMMUNICATION. Communicating verbally and in writing about learning outcomes, thought-building and decision-making. Taking part in debates about issues related to the own field of specialization.
7. THIRD LANGUAGE. Learning a third language, preferably English, to a degree of oral and written fluency that fits in with the future needs of the graduates of each course.
8. TEAMWORK. Being able to work as a team player, either as a member or as a leader. Contributing to projects pragmatically and responsibly, by reaching commitments in accordance to the resources that are available.
9. EFFECTIVE USE OF INFORMATION RESOURCES. Managing the acquisition, structure, analysis and display of information from the own field of specialization. Taking a critical stance with regard to the results obtained.

### Teaching methodology

Lectures, practical work at computer room, exercises and home works.

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### Learning objectives of the subject

This course is an introduction to numerical methods for the solution of partial differential equations, with application to applied sciences, engineering and biosciences.

The course includes the theoretical basis of the Finite Element Method (FEM) for the solution of elliptic and parabolic equations, and an introduction to stabilization techniques for convection-dominated problems, the FEM for compressible flow problems, numerical methods for first-order conservation laws (Finite Volumes, Discontinuous Galerkin) and advanced discretization techniques (such as meshless methods, X-FEM or DG methods).

The course will include frontal lectures and exercises, as well as computer sessions aimed at introducing the bases of the programming of the numerical methods.

### Study load

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|-------------------------------|--------------------|----------|--------|
| Total learning time: 187h 30m | Hours large group: | 60h      | 32.00% |
|                               | Self study:        | 127h 30m | 68.00% |

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### Content

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|---|---|
| <p>Fundamentals of Finite Element Methods (FEM)</p>   | <p>Learning time: 20h<br/>Theory classes: 10h<br/>Laboratory classes: 10h</p> |
| <p>Description:<br/>Basic concepts of the Finite Element Method (FEM) for elliptic and parabolic equations: strong and weak form, discretization, implementation, functional analysis tools, error bounds and convergence, time integration for parabolic equations.<br/>Application to the numerical modelling of flow in porous medium, and potential flow.<br/>Introduction to a posteriori error estimation and adaptivity.<br/>Solution of the convection-diffusion equation. Stabilized formulations for convection dominated problems.</p> |   |
| <p>FEM for incompressible flow problems</p>   | <p>Learning time: 6h<br/>Theory classes: 4h<br/>Practical classes: 2h</p>     |
| <p>Description:<br/>Weak form and discretization of the Stokes equations. Stable FEM discretizations for incompressible flow problems: LBB condition.<br/>Application to microfluidics and geophysics.<br/>Introduction to the numerical solution of the incompressible Navier-Stokes equations.<br/>Introduction to eXtended FEM (X-FEM) for two-phase problems.</p>   |   |
| <p>FEM for wave problems</p>  | <p>Learning time: 10h<br/>Theory classes: 4h<br/>Laboratory classes: 6h</p>   |
| <p>Description:<br/>FEM solution of the 1D wave equation. FEM solution of Helmholtz equation. Non-reflecting boundary conditions.<br/>Application to acoustics.<br/>Introduction to DG for first order conservation laws. Application to acoustics and electromagnetics.</p>  |   |

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| Stochastic FEM  | Learning time: 16h<br>Theory classes: 8h<br>Laboratory classes: 8h |
| Description:<br>Characterization of uncertainty in the input and output of Boundary Value Problems. Non-intrusive and intrusive approaches: Monte-Carlo and Polynomial Chaos. Reducing the stochastic dimension: Karhunen-Loeve expansion. Quick introduction to Reduced Order Models for Uncertainty Quantification. |  |

### Qualification system

Exams (50%) and continuous assesment (exercises, projects and/or oral presentations) (50%).

### Bibliography

#### Basic:

Hughes, Thomas J. R. The finite element method : linear static and dynamic finite element analysis. Englewood Cliffs, NJ: Prentice-Hall International, 1987. ISBN 0133170179.

Wait, R.; Mitchell, A. R. Finite elements analysis and applications. Chichester: John Wiley, 1985. ISBN 0471906778.

Zienkiewicz, O.C.; Taylor, R. L. The finite element method [on line]. 6th ed. Oxford: Butterworth Heinemann, 2005 Available on: <<http://www.sciencedirect.com/science/book/9780750664318>>.

Donea, Jean M; Huerta, A. Finite element methods for flow problems. Chichester: John Wiley & Sons, 2003. ISBN 0471496669.

Ainsworth, M.; Oden, J. T. A posteriori error estimation in finite element analysis. New York: John Wiley & sons, 2000. ISBN 047129411X.

#### Complementary:

Hoffman, Joe D. Numerical methods for engineers and scientists. 2nd ed. rev. and exp. New York: Marcel Dekker, 2001. ISBN 0824704436.

Johnson, Claes. Numerical solution of partial differential equations by the finite element. Mineola, New York: Dover Publications, 2009. ISBN 9780486469003.

Strang, G.; Fix, G. J. An analysis of the finite element method. Englewood Cliffs, NJ: Prentice-Hall, 1973. ISBN 0130329460.

Trefethen, Lloyd N.; Bau, David. Numerical linear algebra. Philadelphia: SIAM, 1997. ISBN 9780898713619.